Math 6140, Fall 2013: Homework # 9.

This assignment covers up to Theorem 42 in the course notes (but not including pages 39–50).

1. (Use Geometer's Sketchpad.) In Figure 1, find an equation relating $\angle 1$, arc ABC and arc DEF.

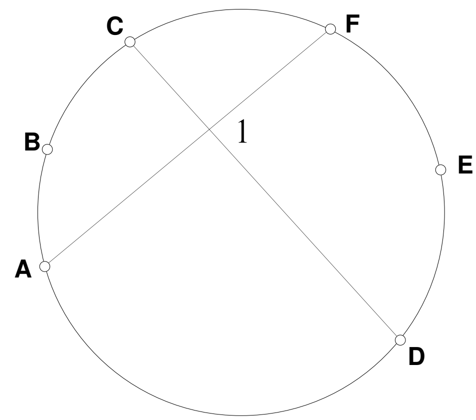

Figure 1

2. (Use Geometer's Sketchpad.) For most quadrilaterals, the four angle bisectors are not concurrent.

   (a) Find an equation that the sides of the quadrilateral have to satisfy if the four angle bisectors are concurrent.

   (b) Make a quadrilateral in which all four sides have different lengths and the four angle bisectors are concurrent. Verify that the equation you found in (a) holds for this example.

3. Given a triangle ABC with orthocenter H, prove that C is the orthocenter of ABH.

4. (In this problem we continue to prove what you discovered in Problem 2 of Assignment 6.) See Figure 2. Given: G is the centroid of $\triangle ABC$, M, N and P are the midpoints of AB, AC and BC respectively, and U, V, W, X, Y and Z are the centroids of the six "little triangles." To prove: the lines UV, XW and GB are concurrent. (Hint: Use a proof similar to that of Theorem 29. You will find it helpful to consider the dashed lines in the picture. You may use Problem 4 of Assignment 7.)

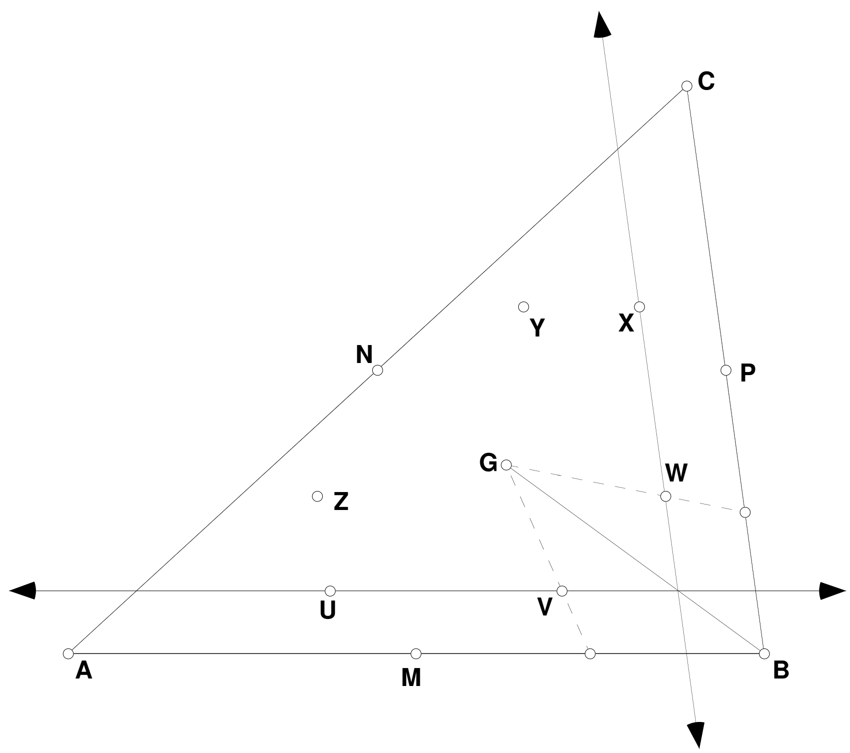

Figure 2

5. (In this problem we continue to prove what you discovered in Problem 2 of Assignment 6.) See Figure 3. Given: G is the centroid of $\triangle ABC$ and U, V, W, X, Y and Z are the centroids of the six "little triangles." To prove: the area of $\triangle DEF$ is 4/9 of the area of $\triangle ABC$. (You may use anything that you have already proved about this picture in Problem 4 of Assignment 7 and the preceding problem. In particular, the lines in Figure 3 that look concurrent are concurrent.)

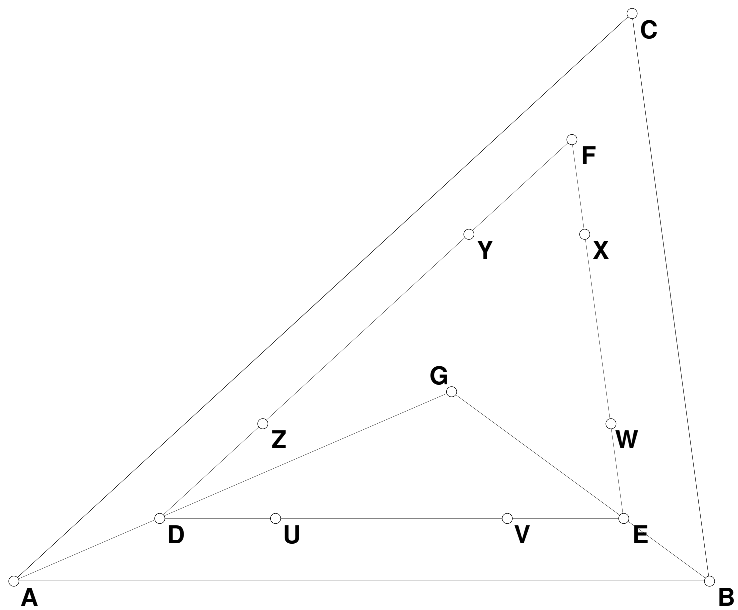

Figure 3

6. (See Figure 4.) Given: AB is parallel to DE, AC is parallel to DF, and BC is parallel to EF. To prove: the lines AD, BE and CF are concurrent. (Hint: use a proof similar to Theorem 29.)

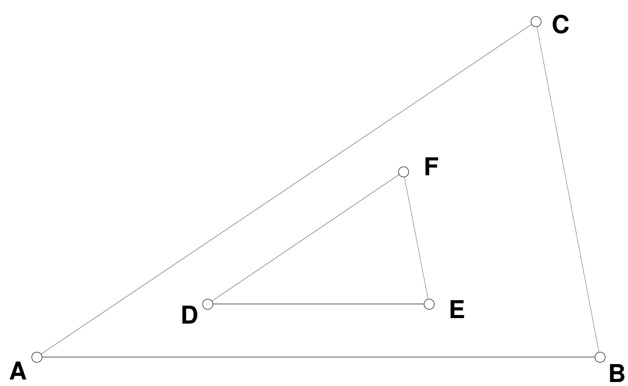

Figure 4

7. (See Figure 5.) Given: Angles ABC and AHG are right angles, and $ABDE$ and $ACFG$ are squares. To prove: $BC = 2HI$.

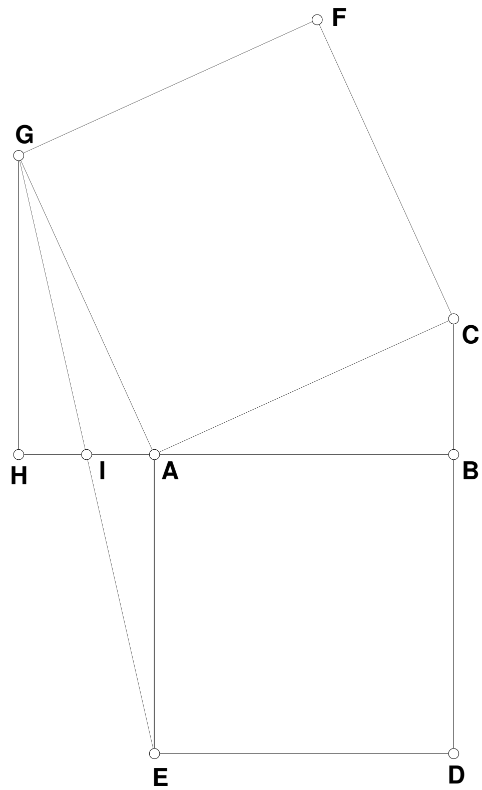

Figure 5

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