Math 6140: Homework # 6.

This assignment covers up to Theorem 31 (but not including pages 39–50). Section 3.3 can be helpful for a couple of these problems.

1. (Use Geometer's Sketchpad.) Construct a triangle and its three medians. The medians divide the triangle into 6 smaller triangles—use a custom tool to construct their centroids (hide the lines you use to do this). Now connect these 6 centroids to form a hexagon. Find the ratio of the area of this hexagon and the area of the original triangle. (Sketchpad will give you a decimal approximation to this ratio: try to find the exact ratio as a fraction. It is **not** 9/25.) Print out a picture.

2. (See Figure 1.) Prove that the bisectors of angles 1, 2 and 3 are concurrent. (Hint: use a strategy similar to the proof of Theorem 26.) Make a picture to illustrate your proof with Geometer's Sketchpad.
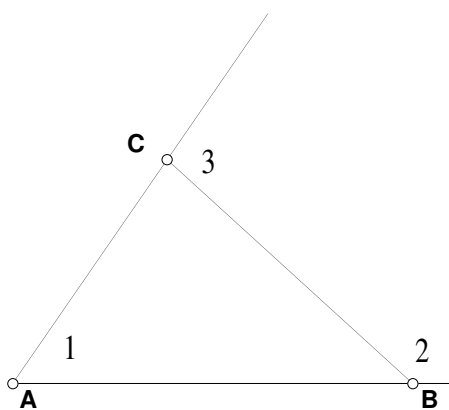
Figure 1

3. The medians of a triangle divide it into 6 small triangles. Prove that they all have the same area.

4. Let ABC be a triangle with centroid G. Let l be the line through G parallel to AB, and let D and E be the points where l intersects AC and BC respectively. Prove that the area of CDE is 4/9 of the area of ABC.

5. (See Figure 2.) Given: M, N and P are the midpoints of AB, AC and BC respectively, MD is parallel to AP, and $MD = AP$. To prove: $CD = NB$. (Hint: there are three parallelograms in this picture. Do *not* draw in any extra lines.)
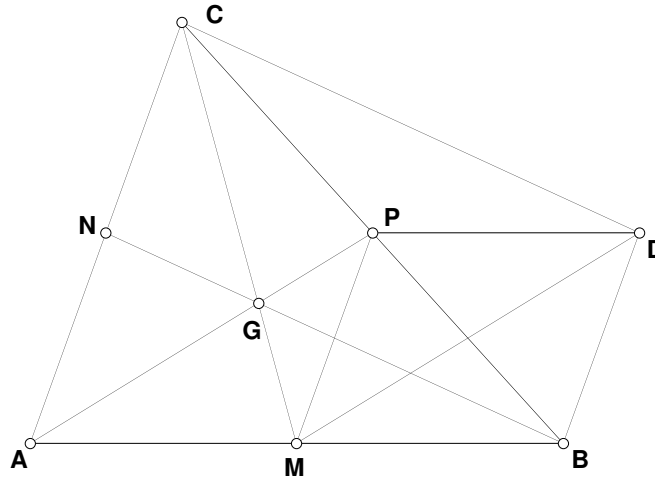

Figure 2

6. (See Figure 3.) Given: $\angle C = \angle D$ and $\triangle APR \cong \triangle BQT$. To prove $\triangle ADF \cong \triangle BCE$
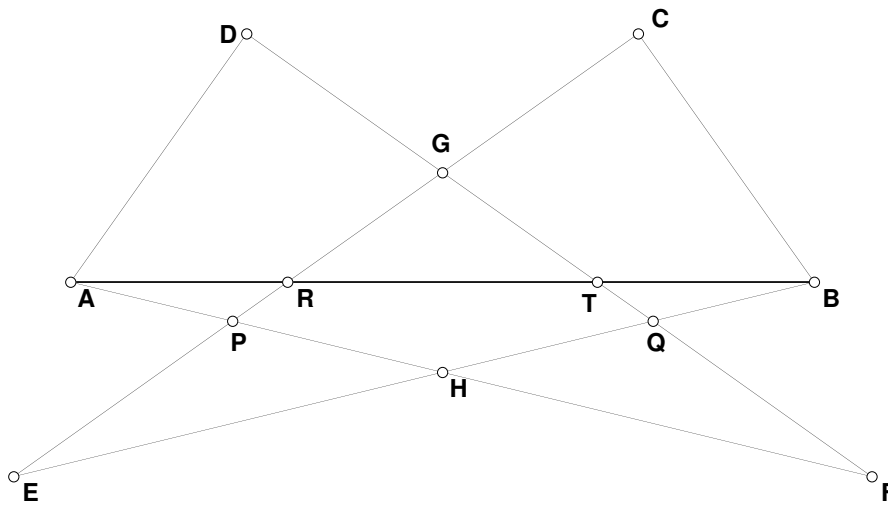

Figure 3

7. (See figure 4.) Prove that
$$\frac{A'B}{A'C}\frac{B'C}{B'A}\frac{C'A}{C'B} = 1.$$

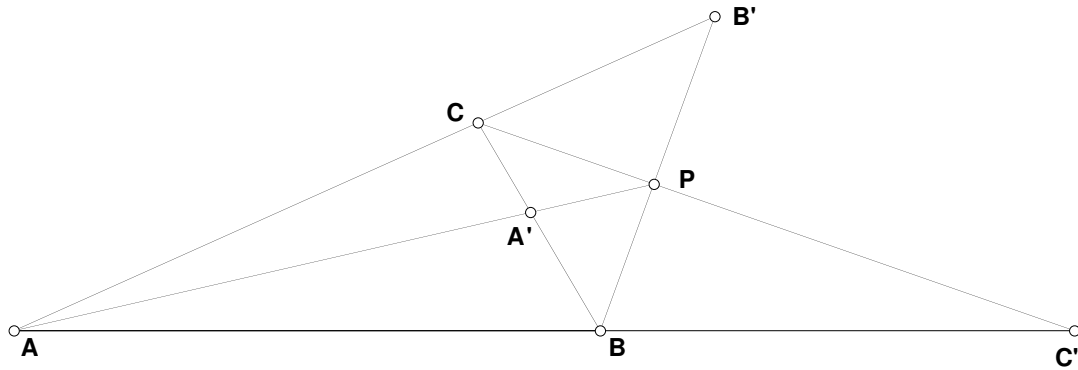(Hint: Use a strategy similar to Problem 3 from Assignment 5.)

Figure 4

—————— The End ——————