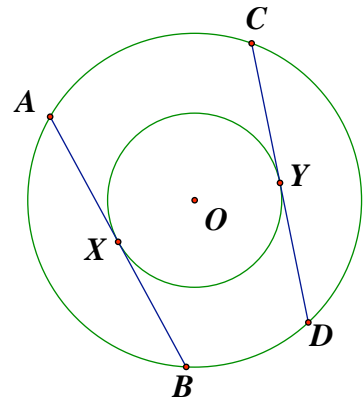
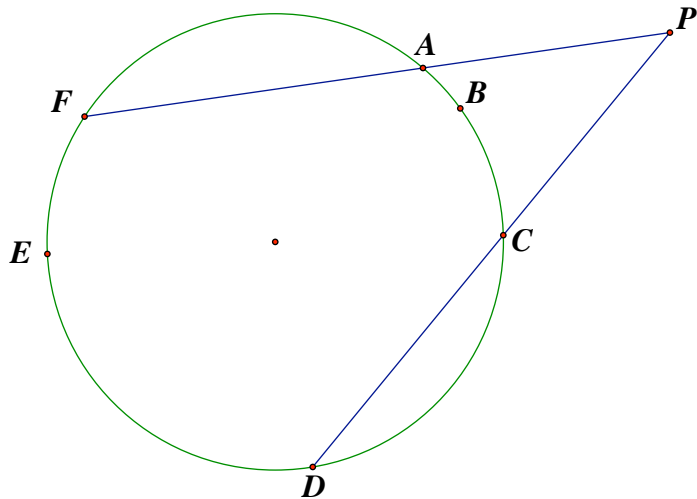


**R. Bruner**  
**Math 6140, Fall 2012, Some possible final exam problems**

1. Prove that in any triangle, the perpendicular bisectors of the sides are concurrent.
2. Prove that any three points which are not collinear lie on a circle.
3. Given:  $O$  is the center of both circles,  $AB$  is tangent to the small circle at  $X$ , and  $CD$  is tangent to the small circle at  $Y$ , show that  $AB = CD$ .

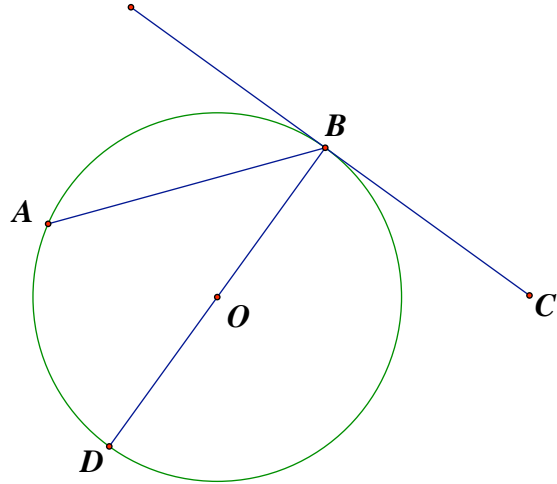


4. Prove that  $\angle FPD = \frac{1}{2}\text{arc } DEF - \frac{1}{2}\text{arc } ABC$ .

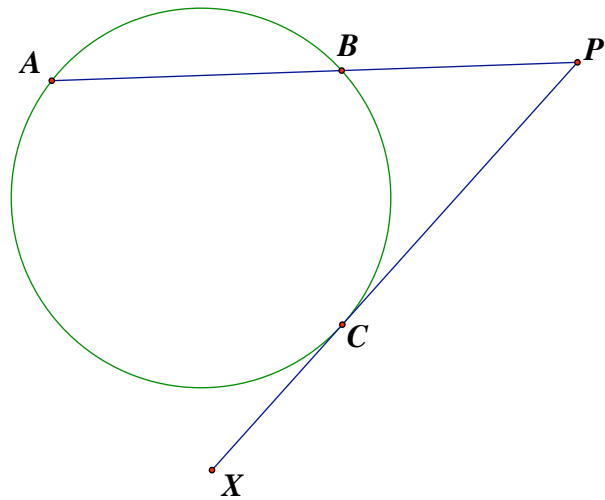


5. In the preceding figure, show that  $PA \cdot PF = PC \cdot PD$ .
6. Let  $H$  be the orthocenter of triangle  $ABC$ . Show that  $A$  is the orthocenter of the triangle  $BCH$ .

7. Given that  $BC$  is tangent to the circle at  $B$ , and that  $BD$  is a diameter of the circle, show that  $\angle ABC = \frac{1}{2} \text{arc } ADB$ .



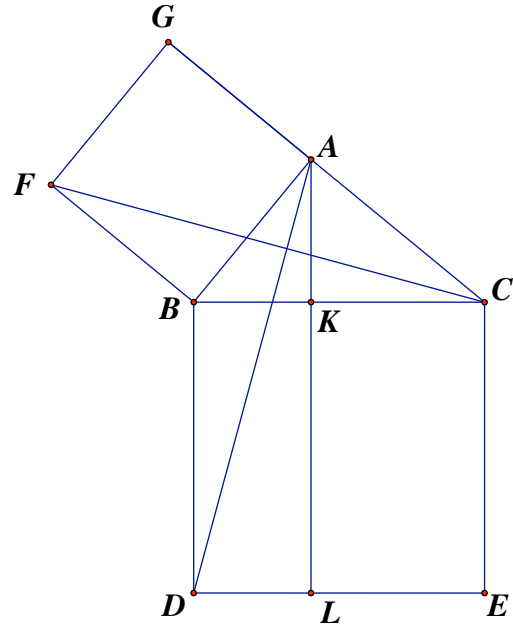
8. Given that the line  $PX$  is tangent to the circle at  $C$ , show that  $PA \cdot PB = PC^2$ .



9. (This is part of Euclid Proposition 47.) Given that

- the lines which look straight are straight,
- $BDEC$  and  $ABFG$  are squares,
- $AL$  is parallel to  $BD$ ,

show, using Euclid, that the area of  $BDLK$  is equal to the area of  $ABFG$ .



10. Prove Euclid's Proposition 20: in any triangle, the sum of any two sides is greater than the third side. You may use anything from Euclid which comes before Proposition 20.
11. Prove Euclid's Proposition 22: to construct a triangle out of three straight lines equal to any three given lines which have the property that the sum of any two is greater than the third. You may use anything from Euclid which comes before Proposition 22.

12. Fill in the possible table with the conclusion you can draw, given the stated comparison between two triangles, the justification using McClure's Notes, and the justification using Euclid. (Note that some entries may be empty, e.g., reasons are not needed if no conclusion is drawn.)

Given	Conclusion	McClure	Euclid
SSS			
SSA			
SAS			
AAS			
ASA			
AAA			

13. Give the definitions and any useful facts you know about

- (a) circumcenter
- (b) incenter
- (c) centroid
- (d) orthocenter

– The End –