Math 6140: Homework # 9.

This assignment covers up to Theorem 42 in the course notes.

1. (Use Geometer's Sketchpad.) In Figure 1, find an equation relating $\angle 1$, arc ABC and arc DEF.

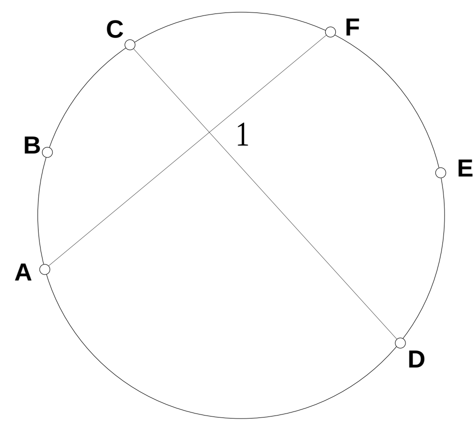

Figure 1

2. (Use Geometer's Sketchpad.) Make a custom tool which constructs the orthocenter of a given triangle (your custom tool should create only the orthocenter, the altitudes should be hidden). Make a triangle ABC and use your custom tool to find the orthocenter. Label the orthocenter H. Then use your custom tool a second time, this time to find the orthocenter of triangle ABH. Indicate on your picture exactly where the orthocenter of triangle ABH is.

3. (In this problem we continue to prove what you discovered in Problem 1 of Assignment 6.) See Figure 2. Given: G is the centroid of $\triangle ABC$, M, N and P are the midpoints of AB, AC and BC respectively, and U, V, W, X, Y and Z are the centroids of the six "little triangles." To prove: the lines UV, XW and GB are concurrent. (Hint: Use a proof similar to that of Theorem 29. You will find it helpful to consider the dashed lines in the picture. You may use Problem 5 of Assignment 7.)

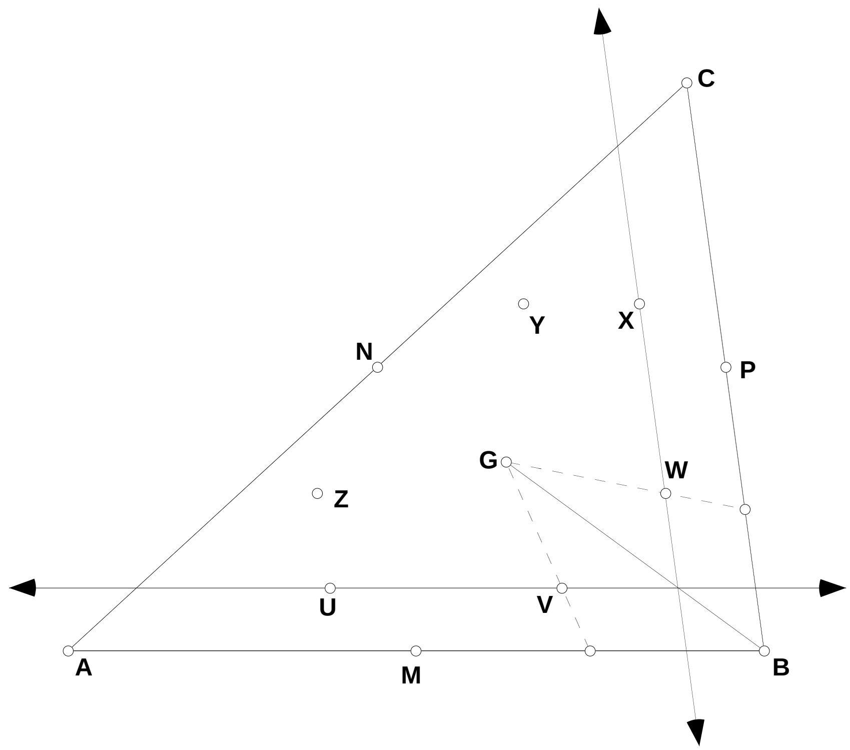

Figure 2

4. (This problem is another step in proving what you discovered in Problem 1 of Assignment 6.) See Figure 3. Given: G is the centroid of $\triangle ABC$ and U, V, W, X, Y and Z are the centroids of the six "little triangles." To prove: the area of $\triangle DEF$ is 4/9 of the area of $\triangle ABC$. (You may use anything that you have already proved about this picture in Problem 5 of Assignment 7 and the preceding problem. In particular, the lines in Figure 3 that look concurrent are concurrent.)

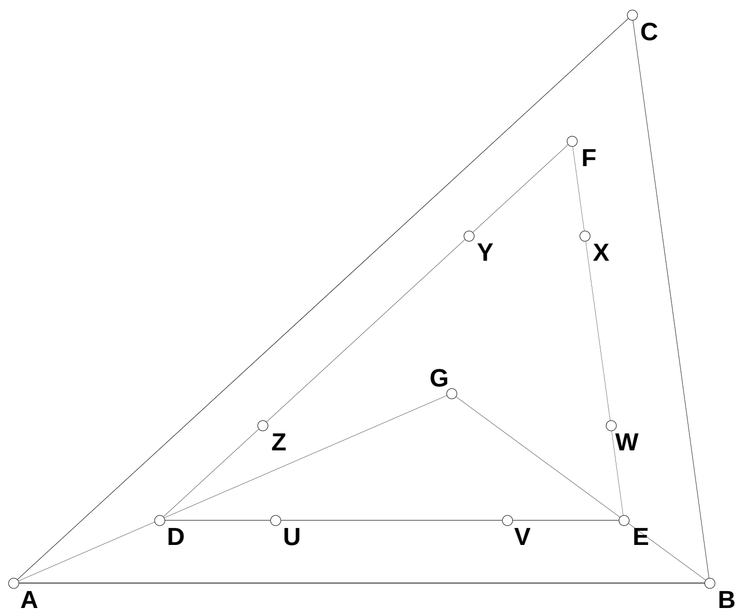

Figure 3

5. (See Figure 4.) Given: AB is parallel to DE, AC is parallel to DF, and BC is parallel to EF. To prove: the lines AD, BE and CF are concurrent. (Hint: use a proof similar to Theorem 29.)

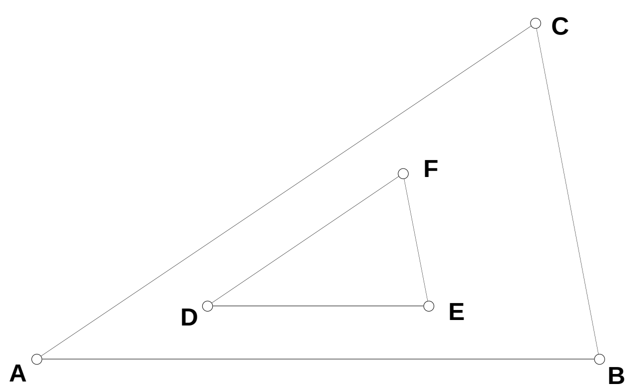

Figure 4

6. Prove the case of Theorem 42 which is illustrated in Figure 42 in the course notes. (Hint: use Theorem 41 as one ingredient.)

7. (See Figure 5.) Given: Angles ABC and AHG are right angles, and $ABDE$ and $ACFG$ are squares. To prove: $BC = 2HI$.

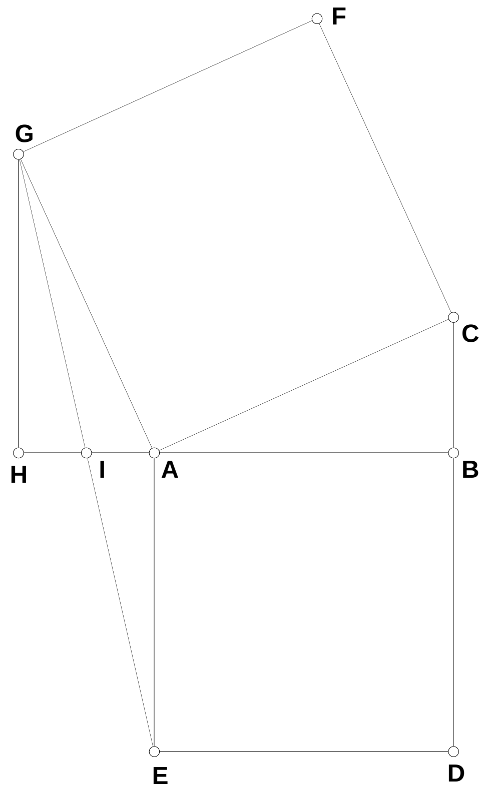

Figure 5

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