Math 6140: Homework # 7.

1. (Use Geometer's Sketchpad.) Start with triangle ABC. Draw a line ℓ that crosses all three lines AB, AC and BC, and let P, Q and R be the intersections of ℓ with the lines AB, AC and BC respectively. Next draw another line m that crosses all three lines AB, AC and BC, and let P', Q' and R' be the intersections of m with AB, AC and BC respectively. Now let X be the intersection of BC and QP', Y the intersection of AC and PR', and Z the intersection of AB and RQ'. What do you notice about the points X, Y, and Z? Print out a picture, then change the position of the lines l and m and print another copy.

2. (Use Geometer's Sketchpad.) Make a picture like Figure 1. Here ABC is an arbitrary triangle, and the things that look like squares are squares. Then find an equation relating the areas of the shaded triangles. Then change the shape of triangle ABC and verify that this equation still holds.

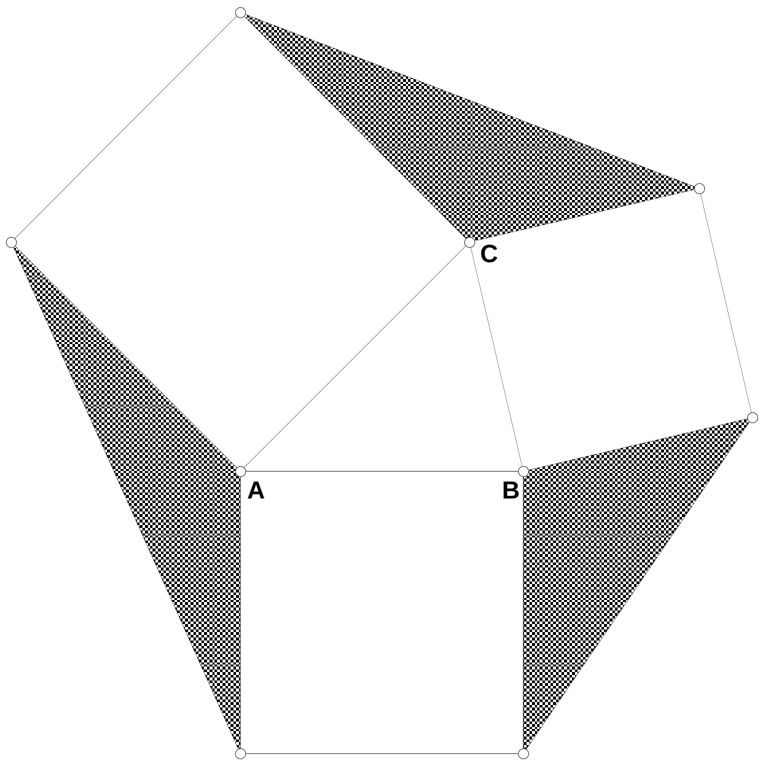

Figure 1

3. (See Figure 2.) Given: M, N and P are the midpoints of AB, AC and BC respectively, MD is parallel to AP, and $MD = AP$. To prove: $CD = NB$. (Hint: there are three parallelograms in this picture. Do *not* draw in any extra lines.)

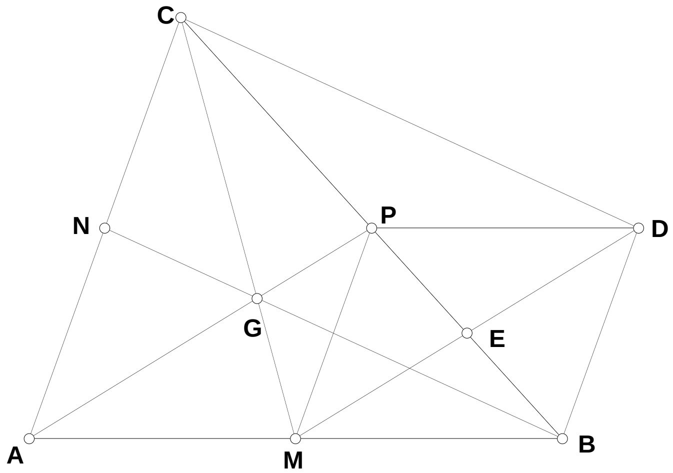

Figure 2

4. (See Figure 2.) Given: M, N and P are the midpoints of AB, AC and BC respectively, MD is parallel to AP, and $MD = AP$.

   (a) Prove that the area of $\triangle CMD$ is twice the area of $\triangle CME$.

   (b) Prove that the area of $\triangle ABC$ is twice the area of $\triangle CMB$.

   (c) Prove that the area of $\triangle CME$ is $3/4$ of the area of $\triangle CMB$.

   Hint: Use Theorem 14 as one ingredient.

5. (In this problem we begin to prove the fact that you discovered in Problem 1 of Assignment 6.) See Figure 3. Given: M, N and P are the midpoints of AB, AC and BC respectively, and U, V, W, X, Y and Z are the centroids of the six "little triangles." To prove: UV is parallel to AB.

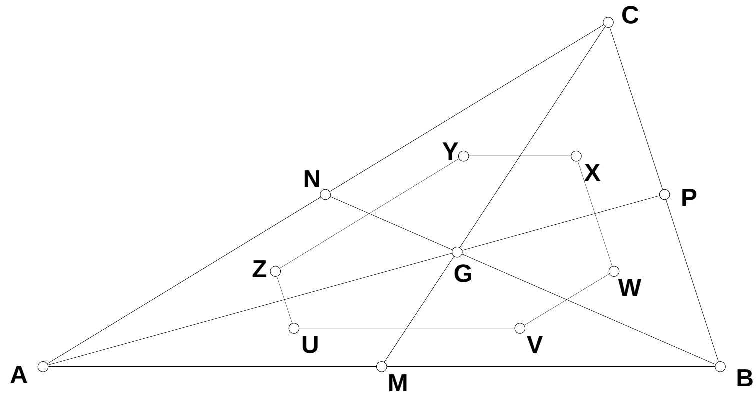

Figure 3

6. (See Figure 4.) Given: M, N and P are the midpoints of AB, AC and BC. To prove: $\dfrac{AD}{DB} = \dfrac{1}{2}$.

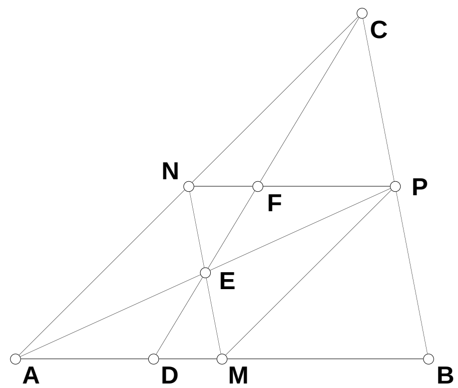

Figure 4

7. (See Figure 5.) Given: DC is parallel to EG, and EG is parallel to AB. To prove: F is the midpoint of EG.

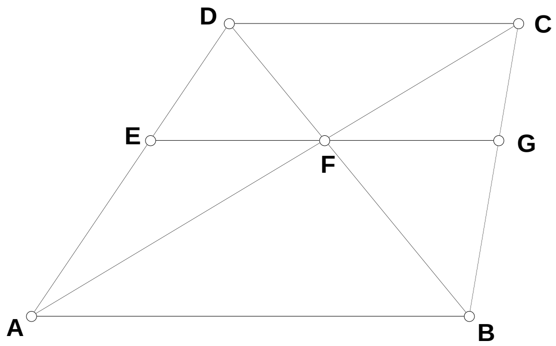

Figure 5

8. (See Figure 6.) Given $ABCD$ is a parallelogram, and BG is parallel to DH. To prove: $DF = BE$.

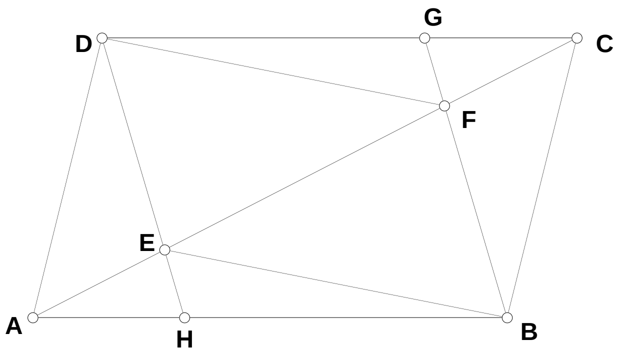

Figure 6

— The End —