Math 6140: Homework # 5.

1. (Use Geometer's Sketchpad.) Draw a triangle ABC. Then draw a line ℓ through A parallel to BC, a line m through B parallel to AC, and a line n through C parallel to AB. Let D be the intersection of ℓ and m, E the intersection of ℓ and n, and F the intersection of m and n. Next, draw in the three altitudes of $\triangle ABC$ and the three perpendicular bisectors of $\triangle DEF$. What do you notice? You do not have to prove anything for this problem.

2. (Use Geometer's Sketchpad.) Construct a quadrilateral $ABCD$ and let M, N, P and Q be the midpoints of the sides. Find the areas of the quadrilaterals $ABCD$, $MNPQ$ and of the four small triangles at the corners of $ABCD$ (use "Triangle Interior" or "Quadrilateral Interior" from the "Construct" menu and then "Area" from the "Measure" menu). Find an equation relating the areas of the four triangles. Print out a picture with the calculations that demonstrate the equation you found. You do not have to prove anything for this problem.

3. (In this problem we prove a fact that you demonstrated experimentally in the previous problem.) Let $ABCD$ be a quadrilateral. Let M, N, P, and Q be the midpoints of the sides. Prove the area of $MNPQ$ is one half the area of $ABCD$.

4. (Use Geometer's Sketchpad.) To say that a quadrilateral is *inscribed* in a circle means that all four of its vertices lie on the circle. Not every quadrilateral can be inscribed in a circle; when a quadrilateral can be inscribed in a circle its angles satisfy a certain equation. Find this equation and print out a copy of the picture with the calculations which demonstrates that the equation holds. You do not have to prove anything for this problem. (Note: for *every* quadrilateral it is true that the sum of the angles is $360°$, so this isn't the equation you're looking for.)

5. (10 points) One of these three statements is very hard to prove. Prove the other two.

   (i) A triangle is isosceles $\Longleftrightarrow$ it has two equal altitudes.

   (ii) A triangle is isosceles $\Longleftrightarrow$ it has two equal angle bisectors.

   (iii) A triangle is isosceles $\Longleftrightarrow$ it has two equal medians.

   Note: since we have defined altitudes, angle bisectors, and medians to be lines, not line segments, the statements require some explanation. In the first statement, "altitude" means the part of the altitude that goes from the vertex to the opposite side, and similarly for the other two statements.

6. (See Figure 1). Prove Case (ii) of Theorem 31. Given: A', B' and C' are collinear. To prove:

$$\frac{A'B}{A'C}\frac{B'C}{B'A}\frac{C'A}{C'B} = 1.$$

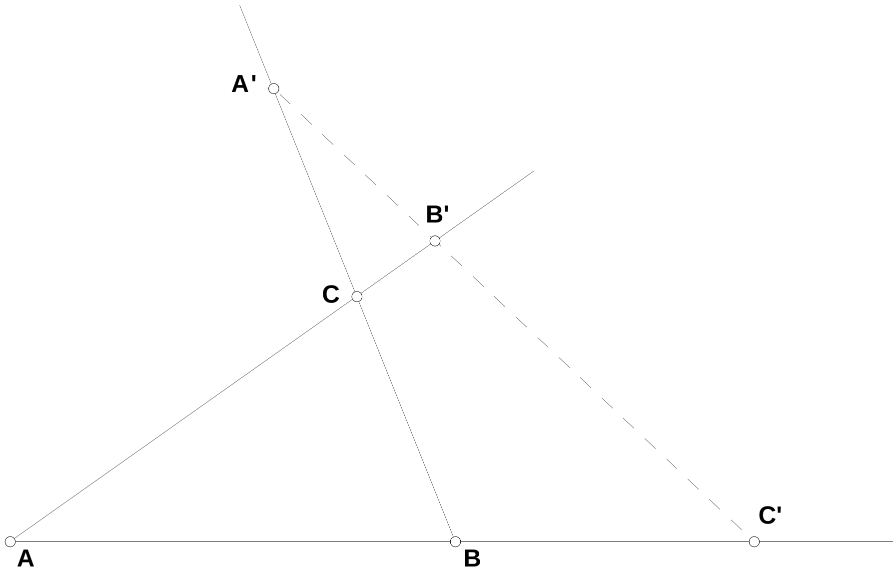

Figure 1

7. (See Figure 5.) Given: O is the center of the circle. To prove: $\angle AOC = 2\angle ABC$.

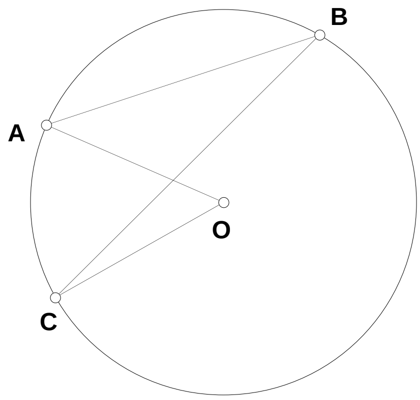

Figure 5

8. (See Figure 2.) Given: L is the midpoint of BC, M is the midpoint of AB, K is the midpoint of AF, and $BE \perp AC$. To prove: $\angle KML$ is a right angle.

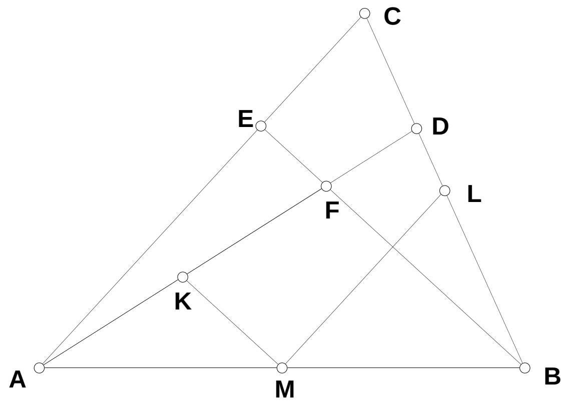

Figure 2

9. (See Figure 3.) Given: $\angle A = \angle B$, $AD = BE$, $\angle ADG = \angle BEF$. To prove: $\angle CFE = \angle CGD$.

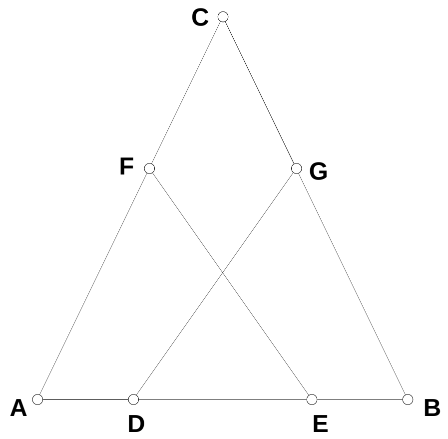

Figure 3

10. (See Figure 4.)

Given: M and N are midpoints, $AQ = QP$, and AD is parallel to BC. To prove: $ABCD$ is a parallelogram.

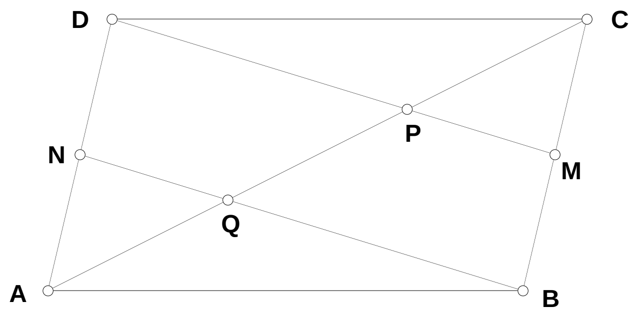

Figure 4