Math 6140: Homework # 3.

This assignment covers up to Theorem 24 in the course notes.

1. (Use Geometer's Sketchpad.) Let ABC be a triangle, Let A' be a point on the segment BC, B' a point on AC, and C' be the intersection of line $A'B'$ with line AB. Find an equation relating the three ratios

$$\frac{A'B}{A'C} \quad \frac{B'C}{B'A} \quad \text{and} \quad \frac{C'A}{C'B}$$

   Print out a picture showing this equation. (Hint: the equation doesn't involve addition or subtraction.) You do not have to prove anything for this problem.

2. (Use Geometer's Sketchpad.) Draw an equilateral triangle ABC using only "Circle by center and radius" and "Segment" from the "Construct" menu. (Hint: a recipe for doing this is on page 241 of the Euclid book.) Measure the height of the triangle. In Problem 3 of Homework # 2 you proved an equation relating the height of $\triangle ABC$ to the distances from P to the three sides of the triangle when P is a point *inside* the triangle. Now find a similar equation that works when P is *outside* the triangle, but *inside* $\angle ABC$. Make sure you say exactly what equation you have in mind. Then verify your equation using the calculator in Geometer's Sketchpad, and make a printout. Then move the point P (staying outside the triangle and inside $\angle ABC$), verify your equation again, and make a second printout. (You do not have to prove anything for this problem.)

3. (See Figure 1.) Given $AE \perp BC$, $BD \perp AC$, and $AF = BF$, prove that $AC = BC$.

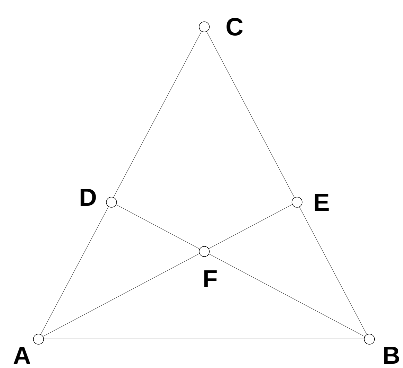

Figure 1

4. Let $ABCD$ be a parallelogram, and let M and N be the midpoints of AB and CD. Prove that MN is parallel to BC.

5. Prove Theorem 20. (Hint: the strategy is like the strategy for Theorem 19.)

6. (see Figure 2). Give the proof of Theorem 24 for Case (ii). Given: M, N, and P are the midpoints of AB, AC, and BC respectively, $MX \perp AB$, and $NX \perp AC$. To prove: PX is the perpendicular bisector of BC. (Hint: Use the same strategy that was used in the course notes for Case (i)).

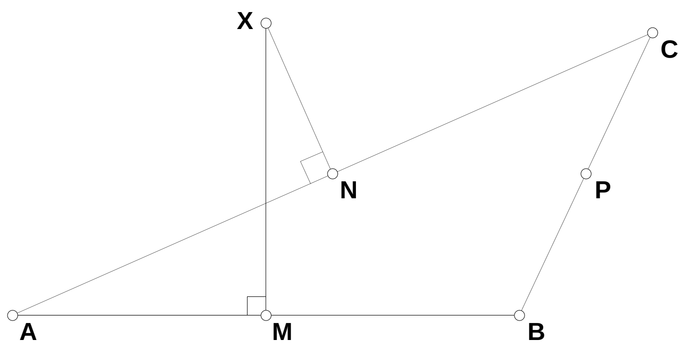

Figure 2

7. Let A and B be two points on a circle. Prove the center of the circle lies on the perpendicular bisector of AB.

8. (15 points) In this problem we prove the fact which was suggested experimentally by Problem 2 of assignment 2. Let ABC be a triangle. Let D and E be points on the segments AC and BC, respectively, with DE parallel to AB. Let F be the intersection of the segments DB and AE, let G be the intersection of AB with the ray CF, and let H be the intersection of DE with ray CF.

   (a) Prove that
   $$\frac{AG}{DH} = \frac{BG}{EH}.$$
   (Hint: Use two pairs of similar triangles.)

   (b) Prove that
   $$\frac{AG}{EH} = \frac{BG}{DH}.$$
   (Hint: Use two other pairs of similar triangles.)

   (c) Use (a) and (b) to show that G is the midpoint of AB.