

R. Bruner
Math 5520, Winter 2010, Midterm Exam
3 March 2010

Each problem with parts is worth 5 points per part, and each problem without parts is worth 10 points, except problems 2(a) and 2(b) are worth 10 points, for a total of 100 points. Read all the problems quickly, and then do the ones you can do most easily first. Please write your answers in your bluebook, and keep the list of questions.

1. Show that $\mathbf{R}^2 - 0$ is connected.
2. Let $f : \mathbf{R}^2 \rightarrow \mathbf{R}$ be a continuous function and $c \in \mathbf{R}$ be a value of f . Let C be the curve where $f(x, y) = c$.
 - (a) Show that C is closed.
 - (b) If c is neither the largest nor the smallest value of f , show that $\mathbf{R}^2 - C$ is not path connected.
3. Show that any continuous function $f : \mathbf{R} \rightarrow \mathbf{R}$ whose values are all integers must be constant.
4. Suppose that $V : S^1 \rightarrow \mathbf{R}^2$ is a continuous vector field which is never 0 on the unit circle. Show that there is an $\epsilon > 0$ such that all the vectors $V(P)$ have length greater than ϵ .
5. Compute the Euler characteristic of the following spaces;
 - (a) the ‘theta’ made by taking the union of the unit circle $x^2 + y^2 = 1$ with the interval $[-1, 1] \times \{0\}$. (That is, the circle together with one of its diameters.)
 - (b) The union of the unit sphere $x^2 + y^2 + z^2 = 1$ and the disk $\{(x, y, z) | x^2 + y^2 \leq 1, z = 0\}$.

– Continued on reverse –

6. Consider the vector field $V(x, y) = (y - x^2 - 1, y - 2)$ on \mathbf{R}^2 .
- (a) Find the nullclines and the direction of flow (NE, NW, SE, SW) in each of the regions into which they divide the plane.
 - (b) Find the critical points of the vector field.
 - (c) Find the index of each critical point.
 - (d) If γ is the circle of radius r about $(1, 0)$, find the winding number $W_\gamma(V)$ as a function of r .
7. Suppose $f : D^2 \rightarrow S^1$ is continuous. Show that there must be a point $P \in S^1$ such that $f(P) = P$. (Hint: recall that S^1 is a subset of D^2 .)
8. Let $\gamma : [0, 1] \rightarrow \mathbf{R}^2$ be a continuous curve. Show that for some $r > 0$ the image of γ lies in the disk of radius r centered at $(0, 0)$.

– The End –