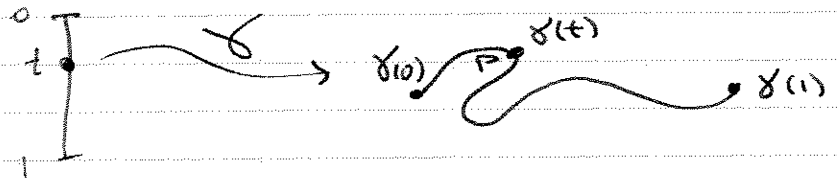
Homework 7 §13 #1, 3, 4 (Homework 8 on next page)

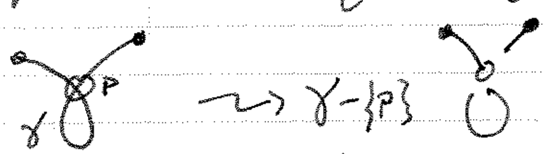
1) $\gamma: I \to \mathbb{R}^2$ is a <u>simple</u> path (no self intersections) and $P = \gamma(t)$, $t \in (0,1)$ is a point of γ which is not an endpoint. Show that $\gamma - P$ is not connected, but has <u>2</u> components.



<u>Proof</u>: γ is a homeomorphism from $[0,1]$ to its image. $[0,1] - \{t\}$ has 2 components $[0,t)$ and $(t,1]$, so the image γ does as well. //

(N.B. In my language, I am using γ to refer to the function in some places, and to its image in others. Context allows complete disambiguation.)

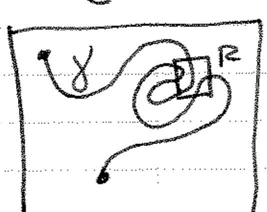
<u>Example</u>: This is false if γ is not simple:



has 3 components

3) $\gamma: I \to I^2$ is a path (i.e. simple curve) in the square. Show that every rectangle in I^2 contains a point not on the path.

<u>Pf</u>:



Suppose $R \subseteq \gamma$. Then if $P \in R$, $P \in \gamma$, and $\gamma - P$ is still connected since $R - P$ is connected and $R - P \subseteq \gamma - P$. (Actually, we have to worry about the possibility that one of the components of $\gamma - P$ contains the whole of $R - P$). Say P is the first point of γ in R. Then just change P to a point that is neither first nor last point of γ in R. //

4) G open, $P \in G$, $G_P = \{Q \in G \mid Q$ can be connected to P by a polygonal chain$\}$. Then G_P is connected and open, and if $Q \notin G_P$ then $G_P \cup \{Q\}$ is not connected.

<u>Pf</u>: G_P is open since any point of G has a disk about it which is in G, any any two points in that disk can be connected by a polygonal chain. So either the whole disk is in G_P or

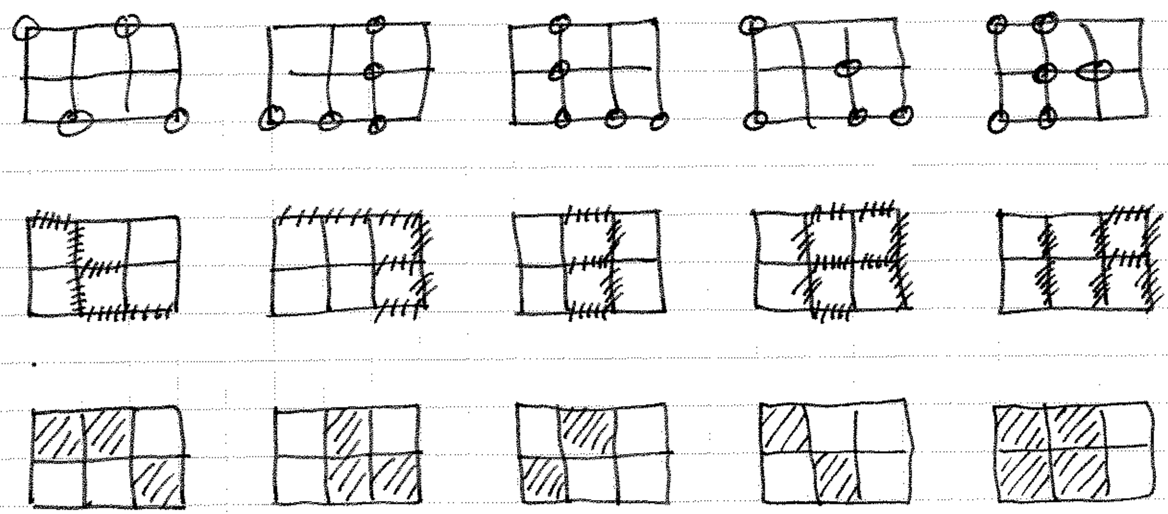
the disk is disjoint from G_p. The former case shows G_p is open and the latter shows $G_p \cup \{Q\}$ is disconnected, since this disk about Q shows Q is not near G_p.

Finally G_p is path connected, hence, connected. //

Homework 8 § 14 # 1,4

(1)

C_1 C_2 C_3 $C_4 = C_1 + C_2$ $C_5 = C_1 + C_2 + C_3$



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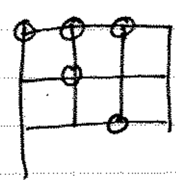
(4) (a) $(k=0)$ $C + C_1 = C_3$

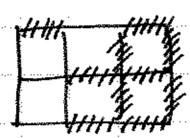
(b) $(k=1)$ $C + C_5 = C_1$

(c) $(k=2)$ $C + C_4 + C_5 = \phi$

(a) $C = C + C_1 + C_1 = C_3 + C_1$



(b) $C = C_1 + C_5 = C_2 + C_3$



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(c) $C = C_4 + C_5 = C_3$