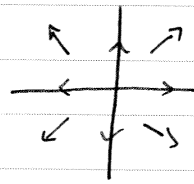
HW 5   m 5520   W'10

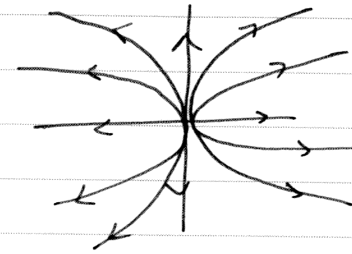
§5 # 2 d, f, g   §6, # 2, 4

(2) (d) $V\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}4x\\y\end{pmatrix}$

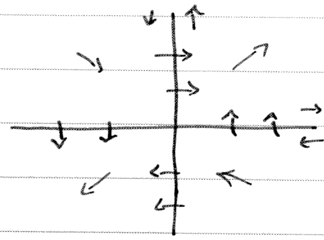


$x = x_0 e^{4t}$
$y = y_0 e^{t}$

$x = \left(\frac{x_0}{y_0^4}\right) y^4$



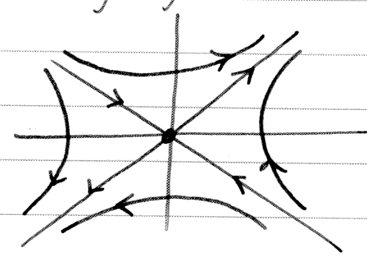
(f) $V\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}y\\x\end{pmatrix}$



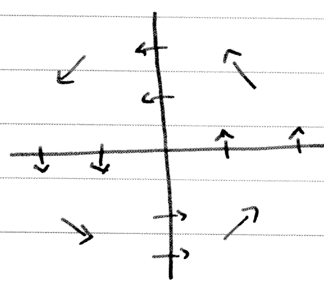
Conservation law: $(x^2-y^2)' = 2xx' - 2yy'$
$= 2xy - 2yx = 0$ so $x^2 - y^2 = C$, constant

$\begin{cases} x'=0 \iff y=0 \\ x'>0 \iff y>0 \end{cases}$

$\begin{cases} y'=0 \iff x=0 \\ y'>0 \iff x>0 \end{cases}$



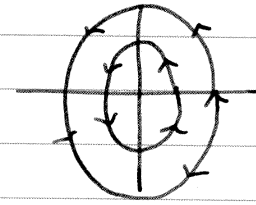
(g) $V\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}-y\\4x\end{pmatrix}$



Conservation Law $\left[(2x)^2 + y^2\right]'$
$= 8xx' + 2yy' = -8xy + 8xy = 0$
So $(2x)^2 + y^2 = $ constant

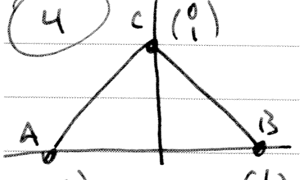
$\begin{cases} x'=0 \iff y=0 \\ x'>0 \iff y<0 \end{cases}$

$\begin{cases} y'=0 \iff x=0 \\ y'>0 \iff x>0 \end{cases}$



§6 (2) $\mathbb{R} \times 0$ does not have the F.P. Property: $f(x) = x+1$ has no F.P.
S^1 does not: $f(\theta) = \theta + \pi/4$ has no f.P.
The annulus
$A = \left\{ \begin{pmatrix}x\\y\end{pmatrix} \,\middle|\, 0 < x^2+y^2 < 1 \right\}$ does not: $f\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}-y\\x\end{pmatrix}$ has no F.P.

(4)

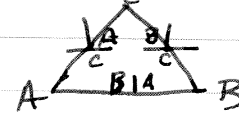


At $\begin{pmatrix}0\\1\end{pmatrix}$, either $f\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}0\\1\end{pmatrix}$ and we're done (f has a F.P.) or $f\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}x\\y\end{pmatrix}$ with $y<1$, in which case $V\begin{pmatrix}0\\1\end{pmatrix}$ points into the lower half plane and Label $\begin{pmatrix}0\\1\end{pmatrix} = C$.

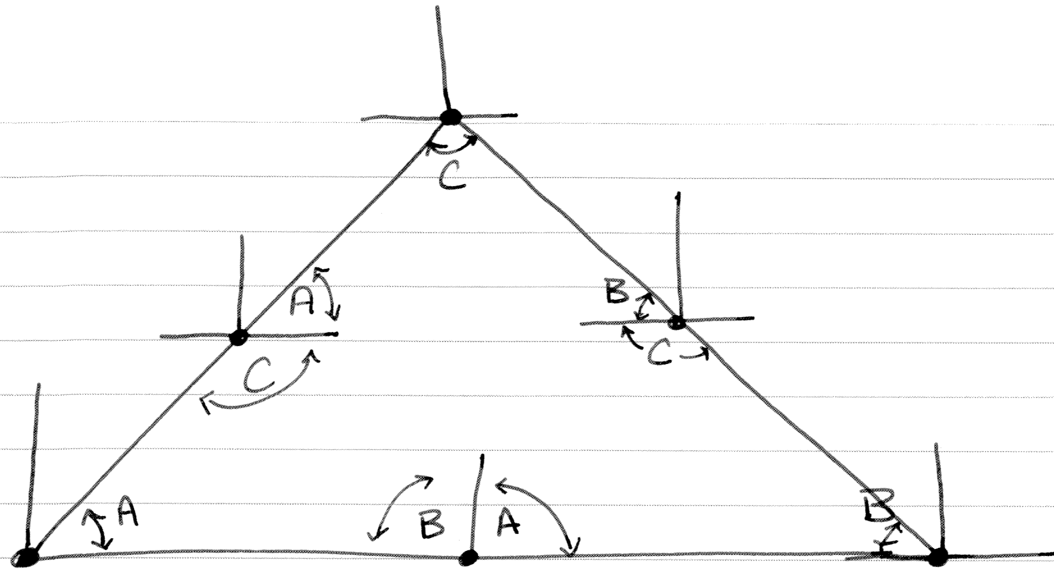
At $\begin{pmatrix}1\\0\end{pmatrix}$, either $f\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}1\\0\end{pmatrix}$ & we're done, or $f\begin{pmatrix}1\\0\end{pmatrix}$ has x-coordinate < 1 so that $V\begin{pmatrix}1\\0\end{pmatrix}$ is in region B.

At $\begin{pmatrix}-1\\0\end{pmatrix}$, $f\begin{pmatrix}-1\\0\end{pmatrix}$ has y-coordinate ≥ 0 and x-coordinate ≥ -1 so Label $\begin{pmatrix}-1\\0\end{pmatrix} = A$.

Along each edge, directions are correct:



Better →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