Homework #12 M 5520 W'10

1. Show that in an abelian group, inverses are unique.

Pf: Suppose that $a+x=0$ and $a+y=0$. Then

$$\begin{aligned} x &= x+0 \\ &= x+(a+y) \\ &= (x+a)+y \\ &= (a+x)+y \\ &= 0+y \\ &= y+0 \\ &= y. \quad // \end{aligned}$$

2. Show that the group $\mathbb{Z}/6$ is isomorphic to the group $\mathbb{Z}/2 \times \mathbb{Z}/3$.

Proof: Let $f: \mathbb{Z}/6 \to \mathbb{Z}/2 \times \mathbb{Z}/3$ by $f(n) = (n,n)$, or more carefully

$$f([n]_6) = ([n]_2, [n]_3) \quad \text{or} \quad f(n+6\mathbb{Z}) = (n+2\mathbb{Z}, n+3\mathbb{Z}).$$

Then f is well-defined since $n \equiv m \ (6)$ implies $n \equiv m \ (2)$ and $n \equiv m \ (3)$. Further, f is a homomorphism: $f(n+m) = (n+m, n+m) = (n,n)+(m,m) \overset{!}{=} f(n)+f(m)$.

Calculating shows f is onto:

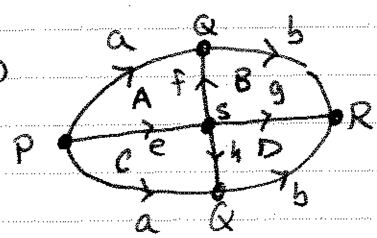
$f(0) = (0,0)$ $\quad$ $f(2) = (0,2)$ $\quad$ $f(4) = (0,1)$

$f(1) = (1,1)$ $\quad$ $f(3) = (1,0)$ $\quad$ $f(5) = (1,2)$

Since both sets have 6 elements, f is also 1-1. //

§23 #1 a,b,c $\quad$ §24 #12

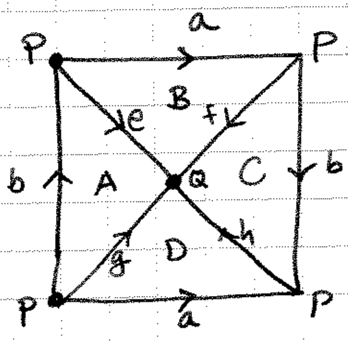
1(a)



	A	B	C	D
a	-1		1	
b		-1		1
e	1		-1	
f	1	-1		
g		1		-1
h			-1	1

	a	b	e	f	g	h
P	-1		-1			
Q	1	-1		1		1
R		1			1	
S			1	-1	-1	-1

1 (b)

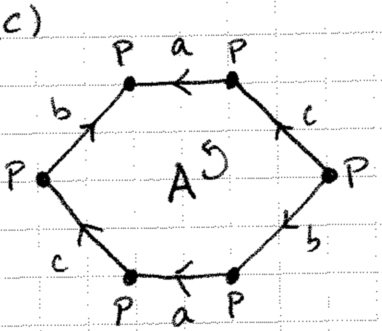


	A	B	C	D
a		-1		1
b	-1		-1	
e	-1	1		
f		-1	1	
g	1			-1
h			-1	1

	a	b	e	f	g	h
P	·	·	-1	-1	-1	-1
Q	·	·	1	1	1	1

Blank entries are 0.

(c)



	A
a	·
b	2
c	·

	a	b	c
P	0	0	0

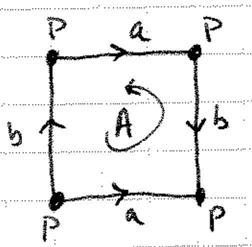
$H_0 = \mathbb{Z} = \langle P \rangle$ $\qquad H_1 = \mathbb{Z}/2 \oplus \mathbb{Z}^2$ $\qquad H_2 = 0$

$= \langle b \rangle \oplus \langle a, c \rangle$

so this must be $P \# P \# P$.

§24 (12)

(a)

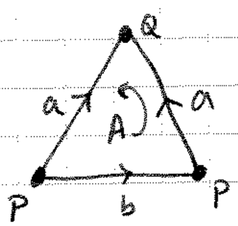


$$C_0 = \langle P \rangle \xleftarrow{[0\ 0]} C_1 = \langle a, b \rangle \xleftarrow{\begin{bmatrix}0\\-2\end{bmatrix}} C_2 = \langle A \rangle$$

$H_0 = \mathbb{Z}$ $\qquad H_1 = \mathbb{Z}/2 \oplus \mathbb{Z}$ $\qquad H_2 = 0$ $\qquad$ (integral)

$H_0 = \mathbb{Z}/2$ $\qquad H_1 = \mathbb{Z}/2 \oplus \mathbb{Z}/2$ $\qquad H_2 = \mathbb{Z}/2$ $\qquad$ (mod 2)

(b)

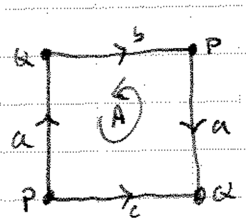


$$C_0 = \langle P, Q \rangle \xleftarrow{\begin{bmatrix}-1&0\\1&0\end{bmatrix}} C_1 = \langle a, b \rangle \xleftarrow{\begin{bmatrix}0\\1\end{bmatrix}} C_2 = \langle A \rangle$$

$H_0 = \mathbb{Z}$ $\qquad H_1 = 0$ $\qquad H_2 = 0$

$\mathbb{Z}/2$ $\qquad 0$ $\qquad 0$

(c)

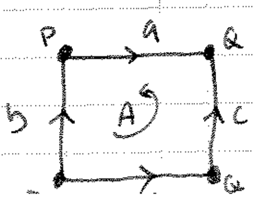


$$C_0 = \langle P, Q \rangle \xleftarrow{\begin{bmatrix}-1&1&1\\1&-1&-1\end{bmatrix}} C_1 = \langle a, b, c \rangle \xleftarrow{[2, 1, -1]} C_2 = \langle A \rangle$$

$H_0 = \mathbb{Z}$ $\qquad H_1 = \mathbb{Z}$ $\qquad H_2 = 0$

$\mathbb{Z}/2$ $\qquad \mathbb{Z}/2$ $\qquad 0$

(d)



$$C_0 = \langle P, Q \rangle \xleftarrow{\begin{bmatrix}-1&0&0\\1&0&0\end{bmatrix}} C_1 = \langle a, b, c \rangle \xleftarrow{[0, 1, -1]} C_2 = \langle A \rangle$$

$H_0 = \mathbb{Z}$ $\qquad H_1 = \mathbb{Z}$ $\qquad H_2 = 0$

$\mathbb{Z}/2$ $\qquad \mathbb{Z}/2$ $\qquad 0$