Solutions to Homework 1

Math 5520 Winter 2010

1. There are many homeomorphisms $(0,1) \to \mathbb{R}$

For example $f(x) = \tan(\pi x - \frac{\pi}{2})$, $f^{-1}(x) = \frac{1}{2} + \frac{1}{\pi}\tan^{-1}(x)$

or $f(x) = \dfrac{x - \frac{1}{2}}{x(x-1)}$, $f^{-1}(x) = \dfrac{1}{x+1+\sqrt{x^2+1}}$.

2. Since $\frac{1}{E} = \frac{1}{a} + \frac{1}{b} - \frac{1}{2}$, we first calculate $1/E$:

$1/E$	$b=3$	$b=4$	$b=5$	$b=6$
$a=3$	$1/6$	$1/12$	$1/30$	0
$a=4$	$1/12$	0	<0	<0
$a=5$	$1/30$	<0	<0	<0
$a=6$	0	<0	<0	<0

Now using $F = \frac{2}{a}E$ and $V = \frac{2}{b}E$ we tabulate (V,E,F) and the type of polyhedron:

(V,E,F)	$b=3$	$b=4$	$b=5$
$a=3$	$(4,6,4)$ tetrahedron	$(6,12,8)$ Octahedron	$(12,30,20)$ Icosahedron
$a=4$	$(8,12,6)$ cube	X	X ← Impossible.
(20, 3	$(20,30,12)$ Dodecahedron	X	X

3. The duality is $(V,E,F) \longleftrightarrow (F,E,V)$: vertices of the dual
$(a,b) \longleftrightarrow (b,a)$
correspond to faces of the original polyhedron. Vertices are connected by an edge if the corresponding faces meet in an edge. So

Tetrahedron $\longleftrightarrow$ Tetrahedron

Cube $\longleftrightarrow$ Octahedron

Dodecahedron $\longleftrightarrow$ Icosahedron

4.

Figure	V	E	F	χ
Annulus	8	-12	$+4$	$=0$
Double annulus	15	-23	$+7$	$=-1$
Sphere	6	-12	$+8$	$=2$
Torus	8	-16	$+8$	$=0$
Möbius strip	6	-9	$+3$	$=0$
"Book"	12	-16	$+5$	$=1$

$\boxed{V - E + F = \chi}$

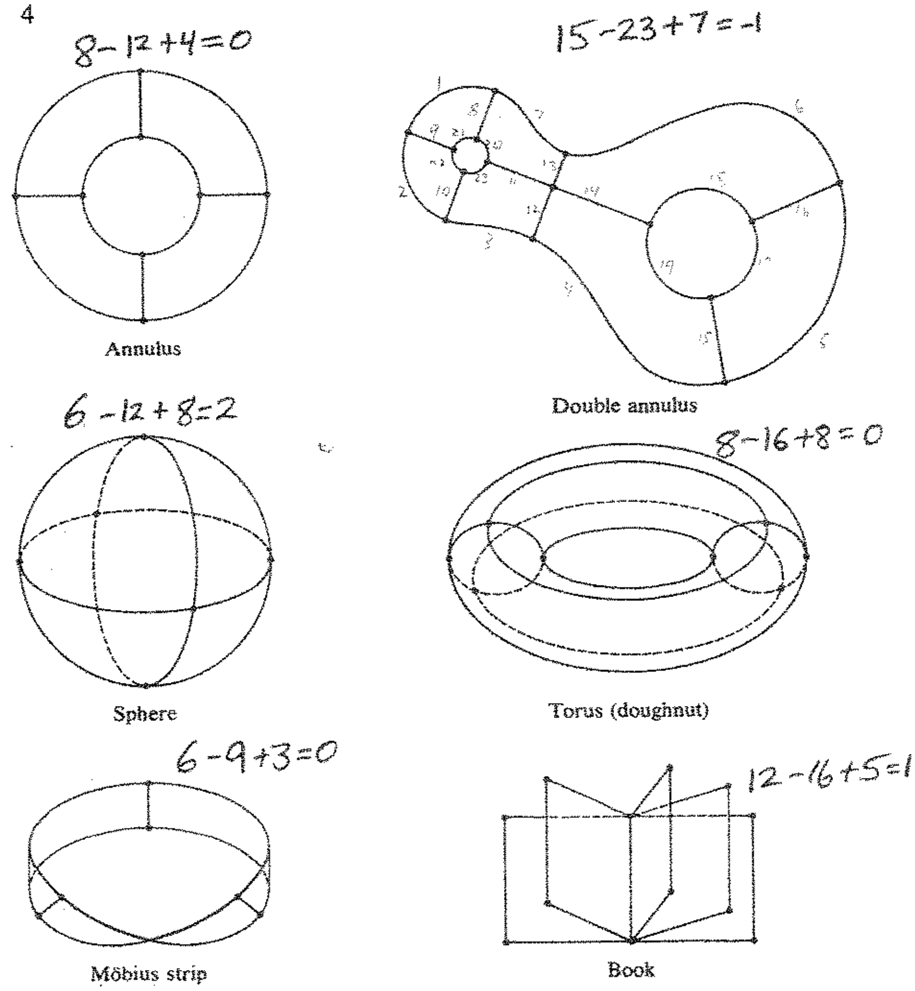

Figure 1.3 Some complexes.

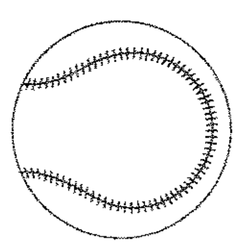

Figure 1.4 Another sphere.