Read all the problems quickly, and then do the ones you can do most easily first.
Please write your answers in your bluebook, and keep the list of questions.
Each problem is worth 20 points, for a total of 200 points, with 40 points possible extra credit.

1. Show that $\mathbf{R}^2$ with any 100 points removed is connected. (Hint: Use the theorems we have proved, not just the definitions.)

2. For each of the following polyhedra:
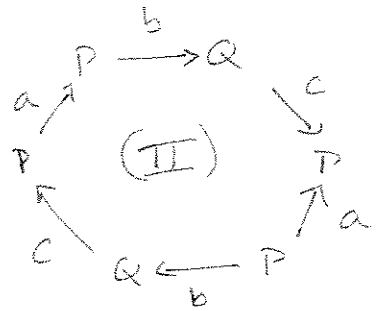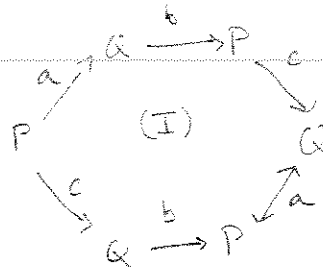
   (a) Compute the Euler characteristic.

   (b) Determine whether or not it is a surface.

   (c) If it is a surface, is it orientable or not?

   (d) If it is a surface, identify it as S, T^g, or P^g. [Hint: use your answers to the preceding questions rather than cut and paste techniques.]

3. Recall that if we sew a disk to a Möbius band, the result is the projective plane P. What do we get if we sew another Möbius band to a Möbius band?

4. Show that there are no continuous functions $f : (-1, 1) \longrightarrow (-1, 0) \cup (0, 1)$ which are onto.

5. Show that there are no continuous functions $f : [-1, 1] \longrightarrow (-1, 1)$ which are onto.

6. Construct a one-to-one continuous function $f : (-4, 1) \cup (1, 4) \longrightarrow (-1, 1)$.

7. Construct a one-to-one continuous function $f : (-4, 4) \longrightarrow [-1, 1]$.

8. Show that a triangulated surface is compact if and only if it has a finite number of triangles.

9. Consider the vector field $V(x, y) = (y - x - 1, xy)$ on $\mathbf{R}^2$.

   (a) Find the nullclines and the direction of flow (NE, NW, SE, SW) in each of the regions into which they divide the plane.

   (b) Find the critical points of the vector field.

   (c) Find the index of each critical point.

   (d) If γ is the circle of radius r about $(1, 0)$, find the winding number $W_v(\gamma)$ as a function of r.

10. In $\mathbf{R}^2$ with the shaded disks below removed, we have the grating shown below. Here are three 1-chains whose boundary is $A_1 + C_3$:

   (a) $A_1A_2 + A_2A_3 + A_3B_3 + B_3C_3$

   (b) $A_1B_1 + B_1B_2 + B_2C_2 + C_2C_3$

   (c) $A_1B_1 + B_1C_1 + C_1C_2 + C_2C_3$

   Two of them are homologous. Which two are they?
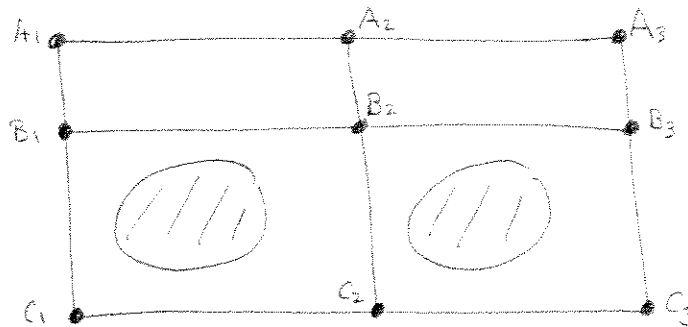
11. (Extra credit) Show that any map $f : D^2 \longrightarrow D^2$ which is the identity on the boundary $S^1 = \partial D^2$ must be onto. [Hint: If f misses a point $p \in D^2$, use it to construct a function $D^2 \longrightarrow D^2$ with no fixed points.]

12. (Extra credit) Let $\gamma : [0, 1] \longrightarrow \mathbf{R}^2$ be a continuous curve which does not intersect the y-axis and which starts at the point $\gamma(0) = (1, 0)$. Show that for some $\epsilon > 0$ the image of γ lies in $\{(x, y) \mid x \geq \epsilon\}$.
   (Hint: consider the composite function $[0, 1] \xrightarrow{\gamma} \mathbf{R}^2 \xrightarrow{\pi} \mathbf{R}$, where π is the projection $\pi(x, y) = x$.)

– The End –