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**Math 5520, Winter 2009, Midterm Exam**
**4 March 2006**

Each problem with parts is worth 5 points per part, and each problem without parts is worth 10 points, for a total of 100 points.

Read all the problems quickly, and then do the ones you can do most easily first. *You may use an earlier problem to solve a later one, whether you have solved the earlier one or not.*

Please write your answers in your bluebook, and keep the list of questions.

1. Show that $\mathbf{R}^2$ with any two points removed is connected. (Hint: Use the theorems we have proved, not just the definitions.)

2. Suppose that $f : X \longrightarrow Y$ is continuous and one-to-one, and that $Y$ is discrete. Show that $X$ is discrete.

3. Show that any continuous function $f : \mathbf{R} \longrightarrow \mathbf{R}$ whose values are all irrational must be constant. (Hint: between any two irrationals there is a rational.)

4. Compute the Euler characteristic of the following spaces.

   (a) the 'figure eight' made by taking the union of the two circles $(x-1)^2 + y^2 = 1$ and $(x+1)^2 + y^2 = 1$.

   (b) the 'dumbbell' consisting of the interval $[-1, 1]$ on the $x$-axis union with the circles of radius 1 centered at $(\pm 2, 0)$.

5. Are the following spaces homeomorphic or not? If not, give a property which distinguishes them, and if so, indicate how a homeomorphism can be constructed.

   (a) $[0, \infty)$ and $\mathbf{R}$.

   (b) $S^1$ and $\mathbf{R}$.

   (c) $(0, 1) \cup (1, 2)$ and $(0, 1) \cup (2, 3)$.

   (d) $\mathbf{R}^2 - \{(0, 0), (1, 0)\}$ and $\mathbf{R}^2 - \{(4, 0), (0, 4)\}$.

– Continued on reverse –

6. Consider the vector field $V(x, y) = (y - x, xy - 1)$ on $\mathbf{R}^2$.

   (a) Find the nullclines and the direction of flow (NE, NW, SE, SW) in each of the regions into which they divide the plane.

   (b) Find the critical points of the vector field.

   (c) Find the index of each critical point.

   (d) If $\gamma$ is the circle of radius $r$ about $(1, 0)$, find the winding number $W_v(\gamma)$ as a function of $r$.

7. Suppose $f : D^2 \longrightarrow [-1, 1]$ is continuous. Show that there must be a point $(x, y) \in D^2$ such that $f(x, y) = x$.
(Hint: we may think of $[-1, 1]$ as a subset of $D^2$ by means of the function $i : [-1, 1] \longrightarrow D^2$ defined by $i(x) = (x, 0)$.)

8. Let $\gamma : [0, 1] \longrightarrow \mathbf{R}^2$ be a continuous curve which does not intersect the $y$-axis and which starts at the point $\gamma(0) = (1, 0)$. Show that for some $\epsilon > 0$ the image of $\gamma$ lies in $\{(x, y) \mid x \geq \epsilon\}$.
(Hint: consider the composite function $[0, 1] \xrightarrow{\gamma} \mathbf{R}^2 \xrightarrow{\pi} \mathbf{R}$, where $\pi$ is the projection $\pi(x, y) = x$.)

– The End –