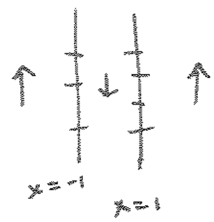
HW 8 §10 # 2 a b c d

§11 # 2

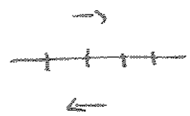
10 ②

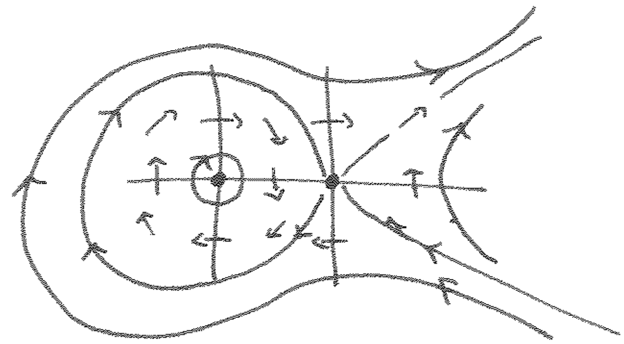
(a) $V\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} y \\ x^2-1 \end{pmatrix}$

Horiz: $x^2 = 1$



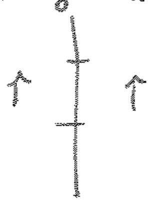
Vert: $y = 0$



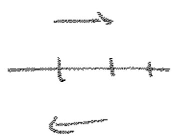


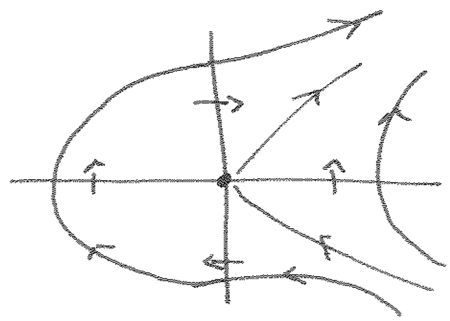
(b) $V\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} y \\ x^2 \end{pmatrix}$

Horiz: $x^2 = 0$



Vert: $y = 0$

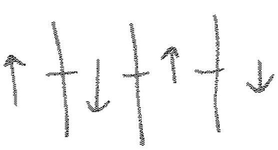




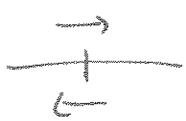
(c) $V\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} y \\ x-x^3 \end{pmatrix}$

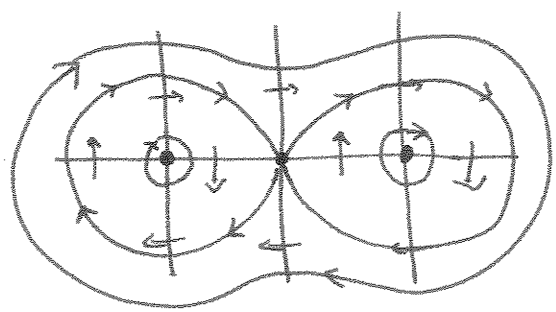
Horiz: $x - x^3 = 0$

$x(1-x^2) = 0$



Vert: $y = 0$

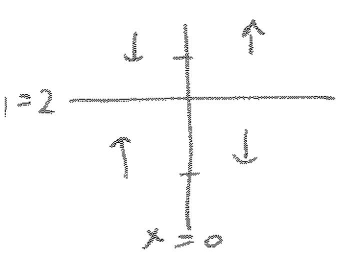




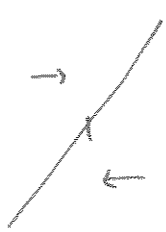
(d) $V\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} y-x \\ xy-2x \end{pmatrix}$

Horiz:

$x(y-2) = 0$



Vert: $y = x$



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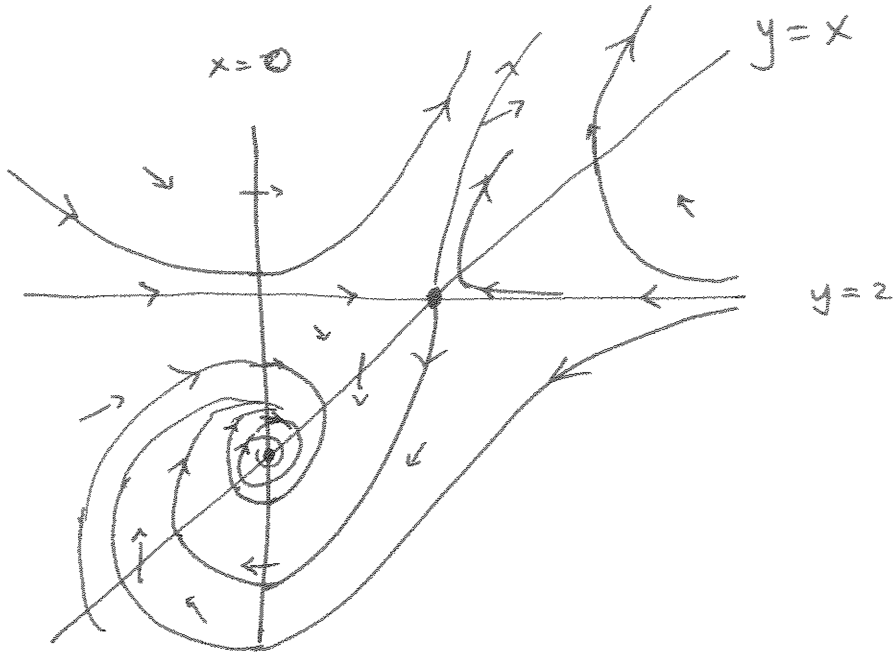


$$J(V) = \begin{pmatrix} -1 & 1 \\ y-2 & x \end{pmatrix}$$

$$J(V)_{\binom{0}{0}} = \begin{pmatrix} -1 & 1 \\ -2 & 0 \end{pmatrix}$$

$\tau = -1$, $\Delta = 2$

$\tau^2 - 4\Delta = -1 - 8 = -9$

spiral focus at $\binom{0}{0}$

$$J(V)_{\binom{2}{2}} = \begin{pmatrix} -1 & 1 \\ 0 & 2 \end{pmatrix}$$

$\tau = 1$, $\Delta = -2$

saddle

$$\lambda_1, \lambda_2 = \frac{1 \pm \sqrt{9}}{2} = \begin{cases} 2 = \lambda_2 \\ -1 = \lambda_1 \end{cases}$$

V_1: $\begin{pmatrix} -1 & 1 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = -\begin{pmatrix} x \\ y \end{pmatrix}$ is $\begin{matrix} -x+y = -x \\ 2y = -y \end{matrix}$

$y = 0$ so $V_1 = \binom{1}{0}$ 

V_2: $\begin{pmatrix} -1 & 1 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = 2\begin{pmatrix} x \\ y \end{pmatrix}$ $\begin{matrix} -x+y = 2x \\ y = 3x \end{matrix}$

§11 #2 Every closed integral path encloses a critical point

Pf: $W_\gamma(V) = 0$ if there are no critical points inside, but for a closed integral path γ, $W_\gamma(V) = 1$. So, there must be a critical point inside such a γ. //

§10 #2c

$V\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} y \\ x - x^3 \end{pmatrix}$ OR $\begin{array}{l} x' = y \\ y' = x - x^3 \end{array}$ so $x'' = x - x^3$

<u>Prop:</u> If $x'' = f(x)$ and $P(x) = -\int f(x)\,dx$ then

$$E = \frac{1}{2}(x')^2 + P(x)$$

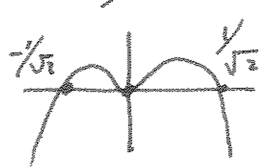
is constant along trajectories. That is, solutions to $x'' = f(x)$ lie in integral curves of E, treating x' as y.

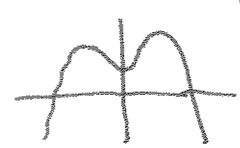
<u>Proof:</u> $\frac{dE}{dt} = \frac{1}{2}(2x'x'') + P'(x)x'$

$$= x'f(x) - f(x)x' = 0. \quad //$$

So solutions to 2(c) lie in the curves $\frac{1}{2}y^2 - \frac{1}{2}x^2 + \frac{1}{4}x^3 = C$, i.e.

$y^2 = x^2 - \frac{1}{2}x^4 + C$. Now $x^2 - \frac{1}{2}x^4 = x^2\left(1 - \frac{x^2}{2}\right)$



So $c<0$ gives  and $c>0$ gives 

Taking square roots (where these values are positive) gives