HW # 7

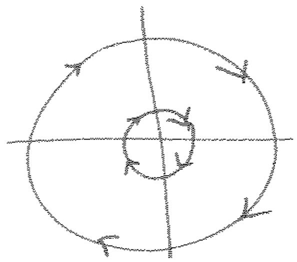
§7 # 1 bcfg, 5 §8 # 1,2

(b) $V\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} y \\ -x \end{pmatrix}$ $\quad \frac{dy}{dx} = \frac{-x}{y}$ OR $y\,dy = -x\,dx$

OR $x\,dx + y\,dy = 0$

OR $d(x^2+y^2) = 0$

2



stable center at $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ 2

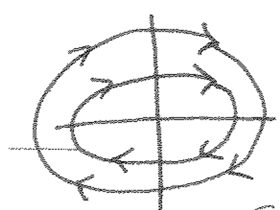
(c) $V\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2y \\ -x \end{pmatrix}$ $\quad \frac{dy}{dx} = \frac{-x}{2y}$ OR $2y\,dy = -x\,dx$

OR $x\,dx + 2y\,dy = 0$

OR $d(x^2+2y^2) = 0$

$d\left(x^2 + \frac{y^2}{(1/\sqrt{2})^2}\right) = 0$

2



Stable center at $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ 2

(f) $V\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x^2-1 \\ 2xy \end{pmatrix}$ $\quad \frac{dy}{dx} = \frac{2xy}{x^2-1}$ OR $(x^2-1)\,dy = 2xy\,dx$

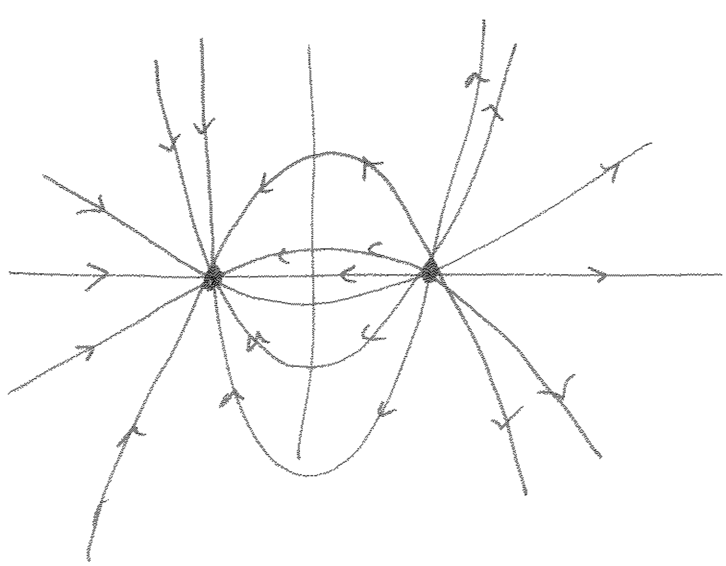
OR $-2xy\,dx + (x^2-1)\,dy = 0$

$\frac{+2x}{x^2-1}\,dx = \frac{1}{y}\,dy \longleftarrow d(\ ?\ ) = 0$

$c + \ln|x^2-1| = \ln|y|$

$y = A(x^2-1)$

3



3

$\begin{pmatrix} \pm 1 \\ 0 \end{pmatrix}$ nodes

$\begin{pmatrix} -1 \\ 0 \end{pmatrix}$ stable

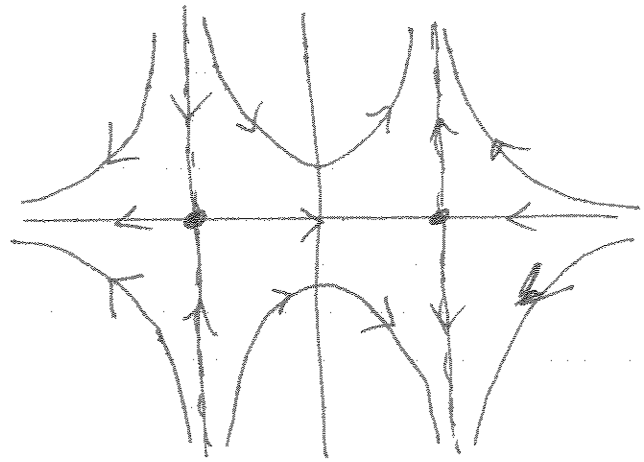
$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ unstable

1) (a) $v\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1-x^2 \\ 2xy \end{pmatrix}$ $\qquad \frac{dy}{dx} = \frac{2xy}{1-x^2}$

$$\frac{1}{y}\,dy = \frac{2x}{1-x^2}\,dx$$

$$\ln|y| = -\ln|1-x^2| + C$$

$$y = \frac{A}{1-x^2}$$

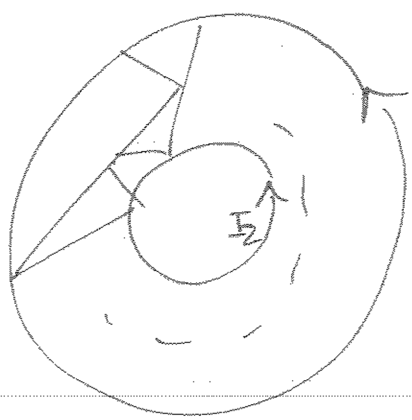


3

$\begin{pmatrix} \pm 1 \\ 0 \end{pmatrix}$ saddles (unstable)

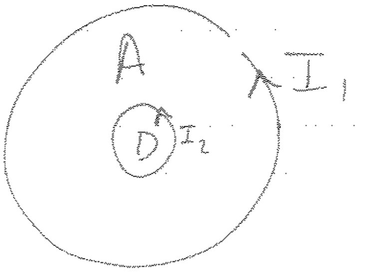
3

5)



$C = I_1 - I_2$

Proof: Triangulate the inner disk. Then

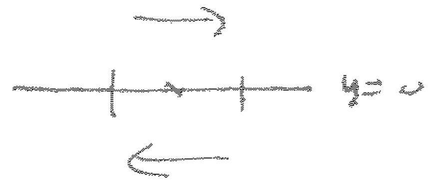


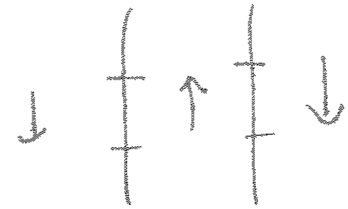
$$I_1 = C(A+D) = C(A) + C(D) = C(A) + I_2$$

so $C(A) = I_1 - I_2$. //

10

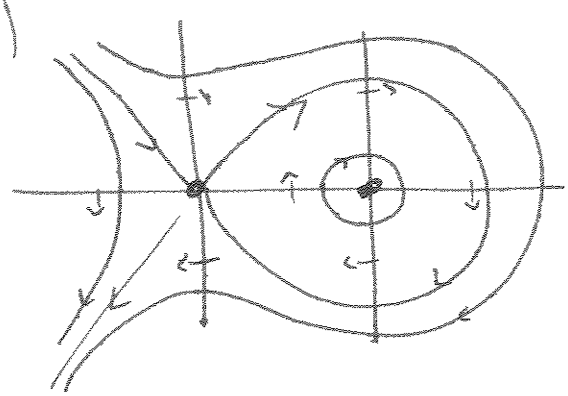
8 (1) $V\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} y \\ 1-x^2 \end{pmatrix}$





$x = \pm 1$

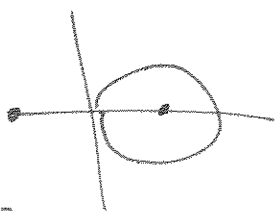
Two C.P. $\begin{pmatrix} \pm 1 \\ 0 \end{pmatrix}$



Index $\begin{pmatrix} -1 \\ 0 \end{pmatrix} = -1$

Index $\begin{pmatrix} 1 \\ 0 \end{pmatrix} = 1$

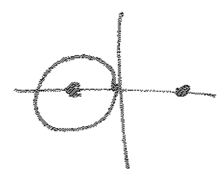
(a)



$W = 1$

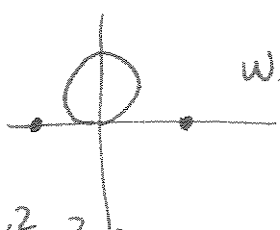
$x^2 - 2x + y^2 = 0$

(b)



$W = -1$

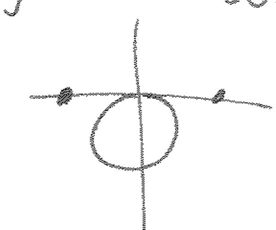
(c)



$W = 0$

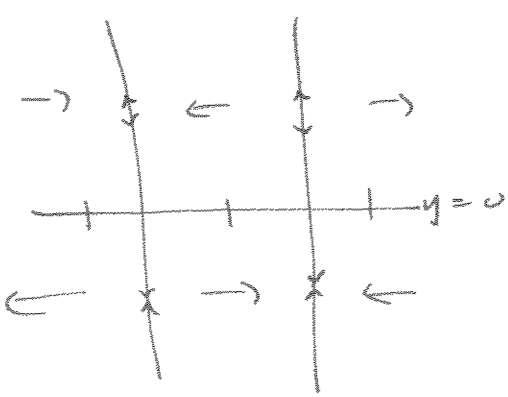
$x^2 + y^2 - 2y = 0$

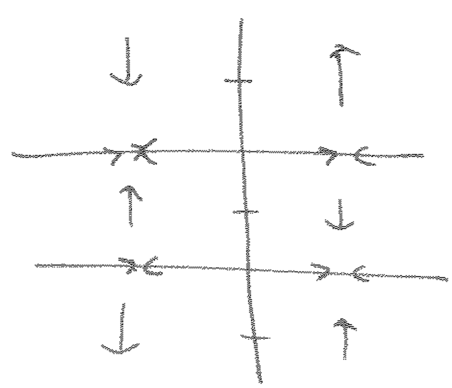
(d) $x^2 + y^2 + 2y = 0$ $W = 0$



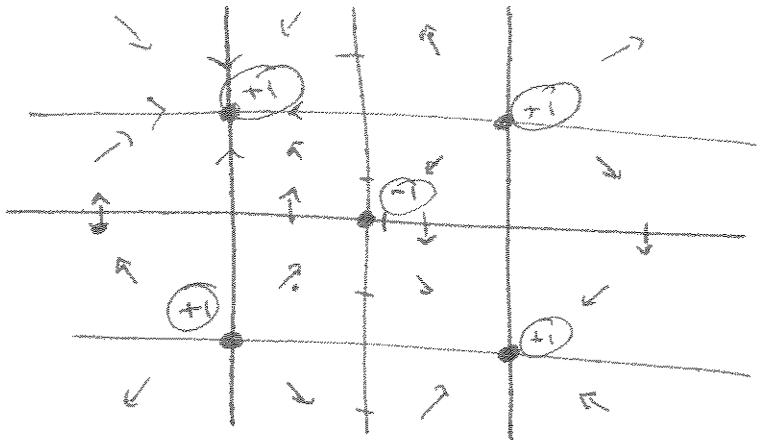
zea = 8

(2) $V\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} y(x^2-1) \\ x(y^2-1) \end{pmatrix}$





2) cont



Indices circled
(Computed by Poincare method)

(a) $x^2+y^2-2x-2y+1=0$
$(x-1)^2+(y-1)^2=1$ $\qquad W = +1$ $\qquad$ (contains (1,1))

(b) $x^2+y^2+x+y=\frac{1}{2}$
$(x+\frac{1}{2})^2+(y+\frac{1}{2})^2=1$ $\qquad W = 0$ $\qquad$ contains (0,0) and (-1,-1)

(c) $x^2+y^2=1$ $\qquad W = -1$ $\qquad$ contains (0,0)

(d) $x^2+y^2=4$ $\qquad W = 3$ $\qquad$ contains all equilibria

2 ea
= 8