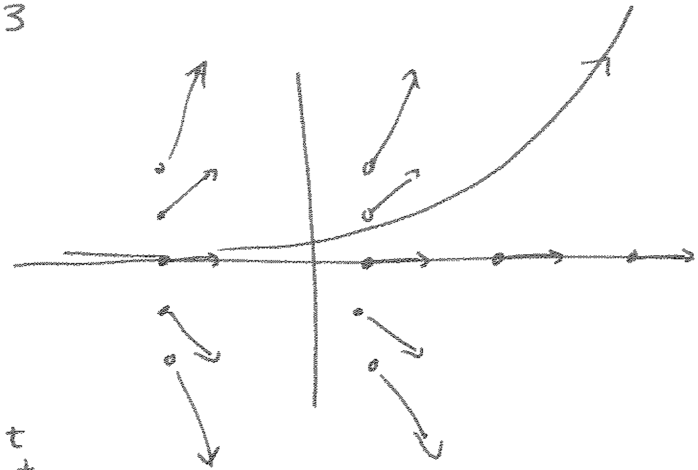
HW # 6 §5 #2 a b c e

§6 # 2,3

§5 2a) $V\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}1\\y\end{pmatrix}$



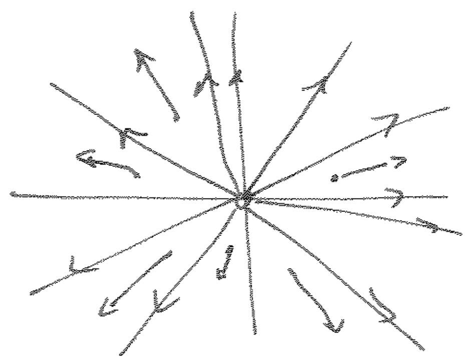
$x' = 1$ so $x = x_0 + t$

$y' = y$ so $y = y_0 e^t$

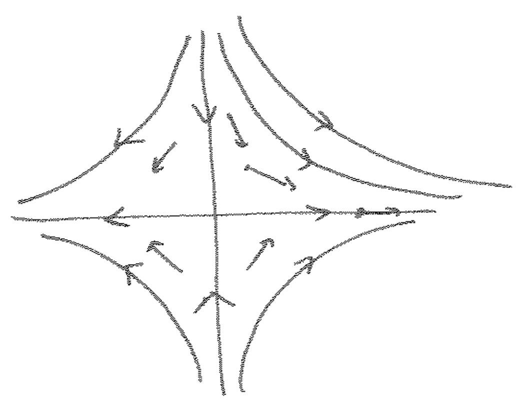
2b) $V\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}x\\y\end{pmatrix}$

$x' = x$ $\quad x = x_0 e^t$

$y' = y$ $\quad y = y_0 e^t$



2c) $V\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}x\\-y\end{pmatrix}$

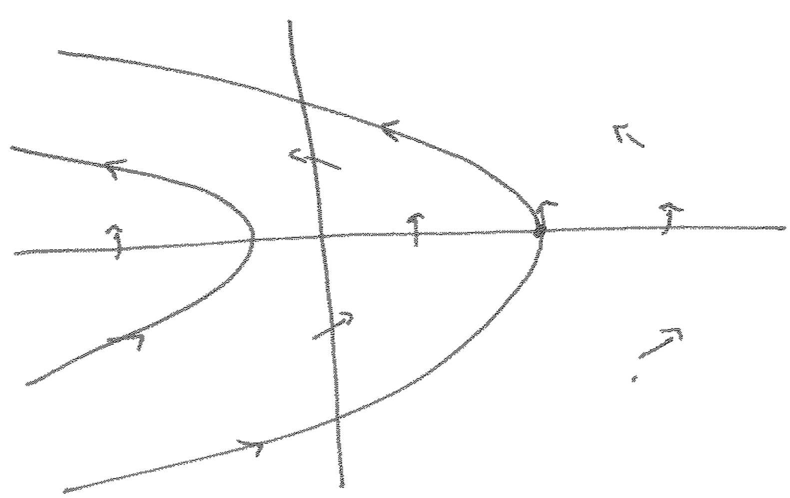


$x' = x$ $\quad x = x_0 e^t$

$y' = -y$ $\quad y = y_0 e^{-t}$

$xy = x_0 y_0$

2e) $V\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}-y\\1\end{pmatrix}$

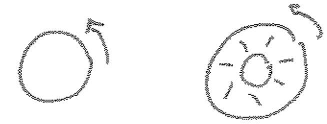


$y' = 1$ so $y = y_0 + t$

$x' = -y_0 - t$ so

$x = x_0 - y_0 t - \frac{t^2}{2}$

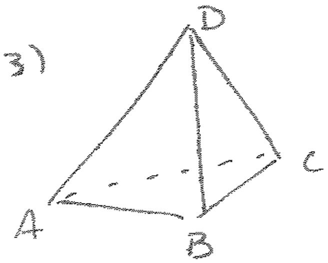
6 (2) S^1 or an annulus can be rotated w/o any fixed points



$\mathbb{R}^2 - 0$ similarly

$\mathbb{R}^2$ or $\mathbb{Z}$ can be shifted $\begin{pmatrix} x \\ y \end{pmatrix} \longmapsto \begin{pmatrix} x+1 \\ y \end{pmatrix}$ or $n \longmapsto n+1$

(3)



let $a = \#$ ABCA, ABCB or ABCC tetrahedra

$b = \#$ ABCD tetrahedra

$c = \#$ ABC faces inside

$d = \#$ ABC faces on the boundary.

Then d is odd by Sperner's Lemma, since this is the number of complete triangles in a Sperner labelling of the ABC face of the tetrahedron.

Now

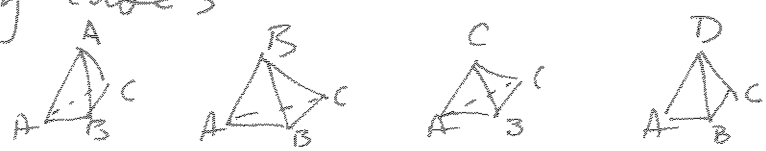
$$\sum_{\text{subtetrahedra}} \#\text{ABC faces} = 2c + d \qquad (1)$$

$$= 2a + b \qquad (2)$$

(1) because interior faces belong to two tetrahedra, while boundary faces belong to only one.

(2) by examining cases

tetrahedron:



\# ABC faces 2 2 2 1

Hence $b = 2c - 2a + d$, so must be odd, hence nonzero. //