Homework #3 Math 5520 W'09

§3 #3, 4, 6

3)

	Set	Bounded	Closed	Compact
a)	$D_1(0)$	✓	✓	✓
b)	S^1	✓	✓	✓
c)	$D_1(0) - S^1$	✓		
d)	$\mathbb{R}\times\{0\}$		✓	
(e)	$\mathbb{R}^2 - \mathbb{R}\times\{0\}$			
(f)	$\mathbb{Z}^2$		✓	
(g)	$\{(\frac{1}{n},0) \mid n=1,2,3\ldots\}$	✓		
(h)	$\mathbb{R}^2$		✓	
(i)	ϕ	✓	✓	✓

(6) part 1

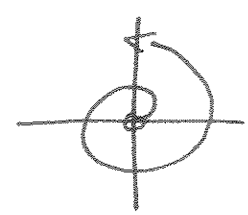
Closure

$D_1(0)$
S^1
$D_1(0)$
$\mathbb{R}\times\{0\}$
$\mathbb{R}^2$
$\mathbb{Z}^2$
add the point $(0,0)$
$\mathbb{R}^2$
ϕ

4) Closed is not topological: $(0,1)$ is not closed in $\mathbb{R}$, but $(0,1)\cong\mathbb{R}$.
Another example: $\mathbb{R}\longrightarrow\mathbb{R}^2$ by $x\longmapsto\begin{pmatrix}e^x\cos x\\ e^x\sin x\end{pmatrix}$

(i.e. $r=e^\theta$ in polar coordinates)

The curve $r=e^\theta$ is homeomorphic to $\mathbb{R}$, but it is not closed since $\binom{0}{0}$ is near the curve but not in it.



Bounded is not topological: $(0,1)\cong\mathbb{R}$.
$(0,1)$ is bounded but $\mathbb{R}$ is not.

6) If $\bar{S}=\{x \mid x\leftarrow S\}$ is the closure of S, then $x\leftarrow\bar{S} \Rightarrow x\in\bar{S}$. That is, $x\leftarrow\bar{S}\Rightarrow x\leftarrow S$.

Proof: Suppose $x\leftarrow\bar{S}$. For any nhood N of x there is a point $y\in N\cap\bar{S}$. Since $y\in\bar{S}$, $y\leftarrow S$. Since $y\in N$, N is a nhood of y. Therefore, there is a point $z\in N\cap S$. That is, every nhood N of x contains a point $z\in S$. Hence $x\leftarrow S$, and so $x\in\bar{S}$. //