Math 5520 W'09 HW 2 Solutions

§2 # 1,3,4,5,6

		set	near points
1)	(a)	S^1	S^1
	(b)	$(0,1)\times\{0\}$	$[0,1]\times\{0\}$
	(c)	$\mathbb{R}\times\{0\}$	$\mathbb{R}\times\{0\}$
	(d)	$\mathbb{Q}\times\{0\}$	$\mathbb{R}\times\{0\}$
	(e)	$\mathbb{Q}\times\mathbb{Q}=\mathbb{Q}^2$	$\mathbb{R}\times\mathbb{R}=\mathbb{R}^2$
	(f)	$\{\binom{x}{y} \mid xy=1\}$	$\{\binom{x}{y} \mid xy=1\}$
	(g)	$\mathbb{Z}^2$	$\mathbb{Z}^2$
	(h)	$\{\binom{r\cos\theta}{r\sin\theta} \mid r\theta=1\}$	$\{\binom{0}{0}\}\cup\{\binom{r\cos\theta}{r\sin\theta} \mid r\theta=1\}$
	(i)	$\emptyset$	$\emptyset$

3) If A has a point near it but not in it, then A is infinite.

Proof 1: If A were finite, say $A=\{a_1,a_2,\dots,a_k\}$, let $d=\min\limits_{1\le i\le k} d(P,a_i)$, for any $P\notin A$. Then $N_{d/2}(P)\cap A=\emptyset$ so $P\not\leftarrow A$. So $P\notin A \Rightarrow P\not\leftarrow A$. //

Proof 2: Suppose $P\notin A$ and $P\leftarrow A$. Then $N_1(P)$ contains a point a_1 of A. Let $\varepsilon_2=\frac{1}{2}d(P,a_1)$. Then $N_{\varepsilon_2}(P)$ contains a point a_2 of A, and $a_2\neq a_1$. Let $\varepsilon_3=\frac{1}{2}\ d(P,a_2)$. Then $N_{\varepsilon_3}(P)$ contains a point a_3 of A, and $a_3\neq a_2$, and $a_3\neq a_1$, similarly. By induction we obtain a sequence of points $a_1,a_2,\dots$ in A all of which are different. Hence A is infinite. //

4) If $P\leftarrow A$ or $P\leftarrow B$ then $P\leftarrow A\cup B$

Proof: If N is a nhood of P it contains a point of A (if $P\leftarrow A$) or a point of B (if $P\leftarrow B$). Hence it contains a point of $A\cup B$. //

If $P\leftarrow A\cup B$ then $P\leftarrow A$ or $P\leftarrow B$.

Proof: Suppose $P\leftarrow A\cup B$ and $P\not\leftarrow A$. Then some nhood N_0 of P contains no points of A. Given any nhood N of P, there is a nhood N_1 of P inside $N\cap N_0$. There is a point of $A\cup B$ in

N_1. Since $N_1 \subset N_0$ this point cannot be in A, so must be in B. Since $N_1 \subset N$ we have just found a point of B in N. Hence every nhood of P contains a point of B, so $P \leftarrow B$. //

5) If every point of A is near B, then any point near A is also near B.

<u>Proof</u>: Suppose $P \leftarrow A$ and N is a nhood of P. Then N contains a point $a \in A$. But N is also a nhood of a, and $a \leftarrow B$, so N contains a point $b \in B$. //

6) If $P \not\leftarrow A$ then P has a nhood which contains no points of A.

<u>Proof</u>: Otherwise, every nhood of P contains a point of A, which would imply that $P \leftarrow A$. //