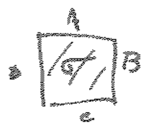
Homework 10 §15 # 1,2,4,6

1) Every k-boundary contains an even # of simplices.

**Lemma**: If $C_1$ and $C_2$ each contain an even number of simplices then $C_1 + C_2$ does also.

**Proof**: Write $C_1 = A_1 + C$ and $C_2 = A_2 + C$ where $C = C_1 \cap C_2$ contains the simplices common to $C_1$ and $C_2$, so that $A_1 \cap A_2 = \emptyset$. Then $C_1 + C_2 = A_1 + A_2$. Now $\# C_1 = 2n_1$ and $\# C_2 = 2n_2$ so $\# A_i = 2n_i - \# C$. Then $\#(C_1 + C_2) = \#(A_1 + A_2) = \# A_1 + \# A_2 = 2n_1 - \# C + 2n_2 - \# C = 2(n_1 + n_2 - \# C)$, which is even. //

**Proof of ①**: If $\sigma$ is a 1-simplex then $\partial\sigma = P + Q$, which has an even # of simplices. If $\sigma$ is a 2-simplex then $\partial\sigma = A + B + C + D$ which also has an even # of simplices.


Now any k-chain is $\sum_{i=1}^{n} \sigma_i$ so $\partial\left(\sum^{n} \sigma_i\right) = \sum^{n} \partial\sigma_i$ is a sum of chains w/ an even number of simplices, so has an even # of simplices. //

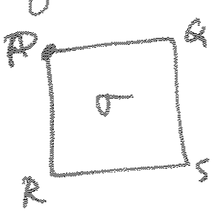
2) The sum of k-cycles is a k-cycle.
The sum of k-boundaries is a k-boundary.

**Pf**: If $\partial\sigma_1 = 0 = \partial\sigma_2$ then $\partial(\sigma_1 + \sigma_2) = \partial\sigma_1 + \partial\sigma_2 = 0 + 0 = 0$.

If $\sigma_1 = \partial A_1$ and $\sigma_2 = \partial A_2$ then $\sigma_1 + \sigma_2 = \partial A_1 + \partial A_2 = \partial(A_1 + A_2)$. //

4) Every boundary is a cycle.

**Proof**: By ② this will be true if it is true for boundaries of individual 2-simplices

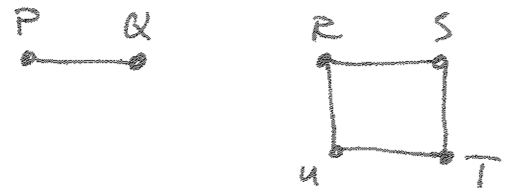



$\partial\sigma = PQ + QS + SR + RP$

$\partial\partial\sigma = P + Q + Q + S + S + R + R + P = 0.$ //

(6) T/F: A 1-chain is a polygonal chain iff its boundary consists of exactly two points.

Proof: False




$$\partial(PQ + RS + ST + TU + RU)$$

$$= P + Q + R + S + S + T + T + U + R + U$$

$$= P + Q$$

but $PQ + RS + ST + TU + RU$ is not a polygonal chain.