4. (a) If $\mathcal{N}$ defined the discrete topology, then one point sets $\{r\}$ would be open. This would require that some $(a,b) \cap \mathbb{Q} \subseteq \{r\}$, but this is impossible because every non-empty interval contains infinitely many rationals.

(b) Let $C \subset \mathbb{Q}$ and $r_1 < r_2$, $r_1 \in \mathbb{Q}$, $r_2 \in \mathbb{Q}$. If $a \in (r_1, r_2)$ is irrational then

$$C = \left[(-\infty, a) \cap C\right] \cup \left[(a, \infty) \cap C\right]$$

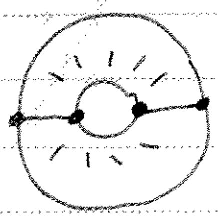
is a disconnection of C, since

- $r_1 \in (-\infty, a) \cap C$, so this is nonempty,
- $r_2 \in (a, \infty) \cap C$, —— " ——.

So C is not connected. Thus, any subset of $\mathbb{Q}$ containing more than one point is disconnected.

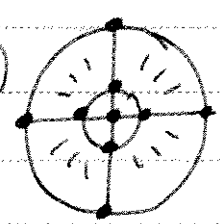
(c) $f(X)$ is a connected subset of $\mathbb{Q}$, so must contain only one element. Hence f is constant.

5. (a)



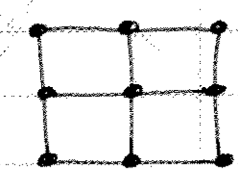
$\chi = 4 - 6 + 2$
$= 0$

(b)



$\chi = 9 - 16 + 4$
$= -3$

(c)



$\chi = 9 - 12 + 0$
$= -3$