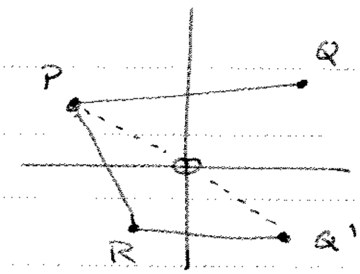
1. If P, Q are two points in $\mathbb{R}^2 - 0$ the straight line $tP + (1-t)Q$ is a connected set containing P and Q, so long as it does not pass through 0. If it does (e.g. P and Q' in the diagram) then the union of paths from P to R and from R to Q gives a connected set containing P and Q.



Thus, every pair of points of $\mathbb{R}^2 - 0$ lies in a connected subset, and hence $\mathbb{R}^2 - 0$ is connected.

2. Suppose $f : \mathbb{R}^2 \to \mathbb{R}$ were a homeomorphism. Then $f : \mathbb{R}^2 - 0 \longrightarrow \mathbb{R} - f(0)$ would also be a homeomorphism. But $\mathbb{R}^2 - 0$ is connected, while $\mathbb{R} - f(0)$ is not, since $\mathbb{R} - f(0) = (-\infty, f(0)) \cup (f(0), \infty)$ is a disconnection.

3. Suppose C_1 and C_2 are compact and let $C = C_1 \cup C_2$. If $(P_1, P_2, P_3, \dots)$ is a sequence in C then infinitely many P_i lie in C_1, or infinitely many P_i lie in C_2, or both. Thus, we have a sequence $(P_{i_1}, P_{i_2}, P_{i_3}, \dots)$ in C_1 or in C_2. By compactness of C_1 and C_2, there is a point $P \in C_1$ (or C_2, resp.) such that $P \leftarrow (P_{i_1}, P_{i_2}, \dots)$ and hence $P \in C$ and $P \leftarrow (P_1, P_2, \dots)$.