7. If X and Y are connected and $X \cap Y \neq \phi$ then $X \cup Y$ is connected.

Proof: Suppose $X \cup Y = A \cup B$ with $A \neq \phi$ and $B \neq \phi$. ~~Then ...~~ and no point of A or B near the other. Then $X = (X \cap A) \cup (X \cap B)$ and no point of $X \cap A$ or $X \cap B$ is near the other. Since X is connected, one of $X \cap A$ and $X \cap B$ must be empty, so either $X \subset A$ or $X \subset B$. We may assume that $X \subset A$. Similarly, $Y \subset A$ or $Y \subset B$ and since A and B are non-empty and $X \subset A$, we must have $Y \subset B$. But then $A \cap B \supseteq X \cap Y \neq \phi$, so each of A and B has a point near the other, a contradiction. Thus, no disconnection of $X \cup Y$ exists, so $X \cup Y$ is connected. //

10. Suppose S has the property that any two points of S lie in a connected subset of S. Then S is connected.

Proof: Suppose $S = A \cup B$ with $A \neq \phi$ and $B \neq \phi$. Pick $P \in A$ and $Q \in B$. Then there is a connected set $C \subseteq S$ with $P, Q \in S$. Since $C = (C \cap A) \cup (C \cap B)$, $C \cap A \neq \phi$, $C \cap B \neq \phi$, there is a point of $C \cap A$ or $C \cap B$ near the other. This is a point of A or B which is near the other. Therefore S is connected. //

12. S is open if each point of S is not near the complement of S.

Prop: If S is open, then $P \in S$ iff P has a nhood contained in S.

Pf: $\Leftarrow$ If $P \in N \subseteq S$ then $P \in S$.

$\Rightarrow$ If $P \in S$, S open, then P is not near $X - S$. So P has a nhood disjoint from $X - S$: $P \in N$ and $N \cap X - S = \phi$. But then $N \subseteq S$, so $P \in N \subseteq S$. //

13 Open in $\mathbb{R}^2$

Not open: a, b, d, f, g.

Open: c, e, h, i.