§3 # 8, 9, 10, 12abdeg

(8) Continuity in the nearness sense $\Rightarrow$ Continuity in the calculus sense

<u>Proof</u> (by contrapositive) If not continuous in the calculus sense then

$\exists\, x_0 \in X$ and $\varepsilon > 0$ such that for all $\delta > 0$ $f(N_\delta(x_0)) \not\subseteq N_\varepsilon(f(x_0))$

Thus, for each $\delta > 0$ there is an $x_\delta \in N_\delta(x_0)$ s.t. $f(x_\delta) \notin N_\varepsilon(f(x_0))$

Then $x_0 \leftarrow \{x_\delta \mid \delta > 0\}$ but $f(x_0) \not\leftarrow \{f(x_\delta) \mid \delta > 0\}$. //

(9) Continuity in the usual sense $\Rightarrow$ Continuity in nearness sense.

<u>Proof</u> (by contrapositive). Suppose $\exists\, P \leftarrow A$ with $f(P) \not\leftarrow f(A)$.

Then $\exists\, \varepsilon > 0$ s.t. $N_\varepsilon(f(P)) \cap f(A) = \phi$, but $\forall \delta > 0$,

$N_\delta(P) \cap A \neq \phi$. Then $f(N_\delta(P)) \cap f(A) \neq \phi$ so

$f(N_\delta(P)) \not\subseteq N_\varepsilon(f(P))$. //

(10) f has an inverse $\Rightarrow$ f is 1-1

<u>Proof</u>: If $gf(x) = x$ for all x then, if $f(P) = f(Q)$ we get $P = gf(P) = gf(Q) = Q$. //

(12) (a) $f\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x+a \\ y+b \end{pmatrix}$ (b) $f\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} (x+1)\cos(\frac{\pi}{2}y) \\ (x+1)\sin(\frac{\pi}{2}y) \end{pmatrix}$

(d) $f\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2(x-y) \\ 2(x+y-1) \end{pmatrix}$ (e) $f\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{4}\begin{pmatrix} x+y+2 \\ x-y+2 \end{pmatrix}$

(g) $f\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} (\frac{3}{2}-x)/(x-1)(x-2) \\ y \end{pmatrix}$