Math 5420, Fall 2015, Test 4
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Read all the problems quickly before starting work.
Turn in your bluebook and keep this list of questions for later reference.

1. For each of the following, decide whether it is
   - a field,
   - an integral domain but not a field, or
   - not an integral domain:

   (a) $\mathbf{Q}[x]/(x^2+1)$
   (b) $\mathbf{Z}_3 \times \mathbf{Z}_5$
   (c) $\mathbf{Z}_3[x]$
   (d) $\mathbf{Z}_3[x]/(x^3+x+1)$
2. Show that a field is an integral domain.
3. Show that if I is a maximal ideal in a commutative ring R then R/I is a field.
4. In the field $\mathbf{Z}_2[x]/(x^3+x+1)$, find the multiplicative inverse of $x+1$.
5. Compute the kernel and image of the (unique) ring homomorphism $\mathbf{Z} \longrightarrow \mathbf{Z}_6 \oplus \mathbf{Z}_9$.

——————— The End ———————