Math 5420, Fall 2015, Test 3
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Read all the problems quickly before starting work.
Turn in your bluebook and keep this list of questions for later reference.

1. Show that there is no one-to-one homomorphism $\phi : \mathbf{Z}_5 \longrightarrow S_4$.

2. Define a one-to-one homomorphism $\phi : \mathbf{Z}_6 \longrightarrow S_n$ for an appropriately chosen n. (Say which value of n you are using.)

3. Show that $\mathbf{Z}_2 \times \mathbf{Z}_6$ is not cyclic.

4. Give an example of a non-normal subgroup and show that it is not normal.

5. Let $G = \langle a \rangle$, the cyclic group generated by a. Show that G/H is also cyclic, for any subgroup H of G.

6. Let H be a subgroup of a group G, and let $a, b \in G$. Show that if $aH \cap bH \neq \emptyset$ then $aH = bH$.

7. (a) (8 points) In $\mathbf{Q}[x]$, compute $\gcd(x^4 - 1, x^7 - 1)$.
   (b) (7 points) Write it as a linear combination of $x^4 - 1$ and $x^7 - 1$.

8. (5 points) Find the remainder you will get after dividing $x^{100} + x^{99} - 1$ by $x - 2$ in $\mathbf{Z}_3[x]$.

9. Let F be a field and let $f(x), g(x), h(x) \in F[x]$. Show that if $f(x) \neq 0$ and $f(x)g(x) = f(x)h(x)$ then $g(x) = h(x)$.

10. Factor $x^5 - x$ into irreducible factors in $\mathbf{Z}_5[x]$.

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