Math 5420, Fall 2015, Test 2
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Read all the problems quickly before starting work.
Turn in your bluebook and keep this list of questions for later reference.

1. (25 points) Express these permutations as products of disjoint cycles and determine their order.

   (a) $(123)(234)$

   (b) $(123)(234)(345)$

   (c) $(1234)(2345)$

   (d) $(1234)(3456)$

   (e) $(1234)(4567)$

2. (10 points) Find all four subgroups of $\mathbf{Z}_6$.

3. (10 points) Find an isomorphism $\mathbf{Z}_2 \times \mathbf{Z}_2 \xrightarrow{\phi} \mathbf{Z}_8^{\times}$.

4. (10 points) Let G be a group of prime order $p = |G|$. Suppose that $a \in G$ and $a \neq e$. Show that $\langle a \rangle = G$.

5. (10 points) Let G be a group of odd order and suppose that $g \in G$ satisfies $g^2 = e$. Show that $g = e$.

6. (10 points) Show that the following pairs of groups are not isomorphic. (In each case, there is a simple property that distinguishes them.)

   (a) S_3 and S_4

   (b) S_3 and $\mathbf{Z}_6$

7. (10 points) Let G be a group.

   (a) Show that if $a \in G$ then $a^2 = e$ if and only if $a = a^{-1}$.

   (b) Show that if $|G|$ is even then there must be an element $a \neq e$ such that $a^2 = e$.

8. Let G be a group, and let $g \in G$. Define

   $$I = \{k \in \mathbf{Z} \mid g^k = e\}.$$

   (a) (10 points) Show that I is closed under addition and subtraction.

   (b) (5 pts) This implies that $I = d\mathbf{Z}$ for some d. What do we call d?

————————— The End —————————