

**Math 5420, Fall 2015, Test 1**  
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Read all the problems quickly before starting work.

Turn in your bluebook and keep this list of questions for later reference.

1. (10 points) Find the gcd (276, 120) and express it as a linear combination of 276 and 120.
2. (10 points) Find all integers  $x$  such that  $9x \equiv 3 \pmod{15}$ .
3. (10 points) Find all integers  $x$  such that both of the following congruences hold:
  - $x \equiv 9 \pmod{10}$  and
  - $x \equiv 10 \pmod{11}$ .
4. (10 points each) Give an example to show that if an integer divides a product, it does not have to divide either factor.
5. (10 points) Let  $f : \mathbf{Z}_{40} \rightarrow \mathbf{Z}_{40}$  be  $f([x]_{40}) = [7x + 3]_{40}$ .
  - (a) Show that  $f$  is one-to-one.
  - (b) Show that  $f$  is onto.
6. (10 points) Explain why the formula  $f([2x]) = [3x]$  defines a well-defined function  $\mathbf{Z}_7 \rightarrow \mathbf{Z}_7$  but not  $\mathbf{Z}_8 \rightarrow \mathbf{Z}_8$ .
7. (10 points) Let  $n > 1$  be an integer, and let  $a \in \mathbf{Z}$ . Define

$$I = \{k \in \mathbf{Z} \mid ka \equiv 0 \pmod{n}\}.$$

Show that  $I$  is closed under addition and subtraction.

————— Continued on Reverse —————

8. (20 points) Consider  $\mathbf{Z}_{30}$  and  $\mathbf{Z}_{30}^\times$ .
- (a) Find a positive integer  $k$  such that  $[a]_{30}^k = [1]$  for every  $[a]_{30} \in \mathbf{Z}_{30}^\times$ .
  - (b) How many elements are in  $\mathbf{Z}_{30}^\times$ ?
  - (c) Can you find  $[a]_{30} \in \mathbf{Z}_{30}^\times$  such that the powers of  $[a]_{30}$  give all the elements of  $\mathbf{Z}_{30}^\times$ ?
  - (d) If  $[a]_{30} \in \mathbf{Z}_{30}^\times$  show that  $g : \mathbf{Z}_{30} \longrightarrow \mathbf{Z}_{30}$  by  $g([x]_{30}) = [ax]_{30}$  is a one-to-one correspondence.
9. (10 points) Show that if  $99x \equiv 0 \pmod{n}$  and  $40x \equiv 0 \pmod{n}$  then  $x \equiv 0 \pmod{n}$ .

————— The End —————