Math 5420, Fall 2014, Test 1
R. Bruner
September 18, 2014

Read all the problems quickly before starting work.
Turn in your bluebook and keep this list of questions for later reference.

1. (5 points) Find the gcd $(282, 114)$ and express it as a linear combination of 282 and 114.

2. (10 points) Find all integers x such that $3x - 9$ is divisible by 13.

3. (10 points) Find all integers x such that both of the following congruences hold:
   - $x \equiv 3 \pmod 5$ and
   - $x \equiv 6 \pmod 7$.

4. (10 points) Let $f : \mathbf{Z}_{12} \longrightarrow \mathbf{Z}_{12}$ be $f([x]_{12}) = [7x+3]_{12}$.
   (a) Compute $f \circ f$.
   (b) Is f one-to-one?
   (c) Is f onto?

5. (10 points) Does the formula $f(m/n) = (n-m)/n^2$ give a well-defiined function from the rationals to the rationals? Why or why not?

6. (10 points) Let $n > 1$ be an integer, and let $[a]_n \in \mathbf{Z}_n^{\times}$. That is, $(a, n) = 1$. Define

$$I = \{k \in \mathbf{Z} \mid {[a]_n}^k = [1]_n\}.$$

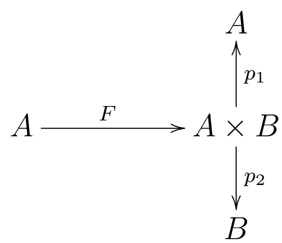
   Show that I is closed under addition and subtraction.

7. (15 points) Consider $\mathbf{Z}_{18}^{\times}$.
   (a) Find a positive integer k such that $[a]_{18}^k = [1]$ for every $[a]_{18} \in \mathbf{Z}_{18}^{\times}$.
   (b) How many elements are in $\mathbf{Z}_{18}^{\times}$?

(c) Can you find $[a]_{18} \in \mathbf{Z}_{18}^{\times}$ such that the powers of $[a]_{18}$ give all the elements of $\mathbf{Z}_{18}^{\times}$?

8. (10 points) Show that if ${[x]_n}^{10} = [1]_n$ and ${[x]_n}^{7} = [1]_n$. then $[x]_n = [1]_n$.

9. (20 points) Suppose that A is a non-empty set and that $f : A \longrightarrow B$ is a function. Define functions F, p_1, and p_2, as shown here



by $F(a) = (a, f(a))$, $p_1(a, b) = a$, and $p_2(a, b) = b$.

(a) Compute $p_1 \circ F$.

(b) Compute $p_2 \circ F$.

(c) Show that F is one-to-one.

(d) Show that p_2 is onto.

—————— The End ——————