MAT 2250 F15 TEST 3

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1. (a) **Yes**, $\mathrm{Nul}\left(\begin{bmatrix} 1 & -2 & 3 \end{bmatrix}\right)$

   (b) **No**: if $\begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix}$ & $\begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix}$ are in there $x_1+x_2-2(y_1+y_2)+3(z_1+z_2)=2$, not 1

   (c) **Yes** $= \mathrm{Span}\left(\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 2 \end{bmatrix}\right)$

   (d) **No** $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ in there $\Rightarrow z-x=2$. Doubling, $z-x=4$, not 2.

   (e) **No** Mult. by a negative scalar

   (f) **No**, Mult. by π or some other noninteger

   (g) **Yes** $\deg(p+q) \le \max(\deg p, \deg(q))$

   (h) **No** $\deg(p+q)$ could be < 3 even if $\deg(p) = 3 = \deg(q)$

   (i) **Yes** $(p+q)(1) = 0+0 = 0$ and $(cp)(1) = c \cdot 0 = 0$.

   (j) **Yes** $(p+q) = xp' + xq' = x(p'+q')$ and $cp = x(cp)'$.

2ea

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2. $$\begin{bmatrix} 1 & 1 & 2 & 1 \\ 1 & 1 & 2 & 1 \\ 2 & 0 & 2 & 1 \\ 3 & 0 & 3 & 1 \\ 1 & 1 & 2 & 1 \end{bmatrix} \begin{matrix} \\ \xrightarrow{-R_1} \\ \xrightarrow{-2R_1} \\ \xrightarrow{-3R_1} \\ \xrightarrow{-R_1} \end{matrix} \begin{bmatrix} 1 & 1 & 2 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & -2 & -2 & -1 \\ 0 & -3 & -3 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix} \xrightarrow{*-1/2} \begin{bmatrix} 1 & 1 & 2 & 1 \\ 0 & 1 & 1 & 1/2 \\ 0 & -3 & -3 & -2 \\ & & & \end{bmatrix} \xrightarrow{+3R_2}$$

$$\begin{array}{cccc} * & * & & * \\ \end{array}$$
$$\begin{bmatrix} \underline{1} & 1 & 2 & 1 \\ 0 & \underline{1} & 1 & 1/2 \\ 0 & 0 & 0 & \underline{-1/2} \\ & & & \end{bmatrix}$$

Omit 3rd vector.
Keep 1st, 2nd and 4th.

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3. $$\begin{bmatrix} 1 & 1 & 1 & 0 \\ 2 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 2 & 1 & 0 & 0 \end{bmatrix} \begin{matrix} \\ \xrightarrow{-2R_1} \\ \xrightarrow{-R_1} \\ \xrightarrow{-R_2} \end{matrix} \begin{bmatrix} \underline{1} & 1 & 1 & 0 \\ 0 & \underline{-1} & -1 & 1 \\ 0 & 0 & \underline{-1} & 0 \\ 0 & 0 & 0 & \underline{-1} \end{bmatrix}$$

pivot in every row, so adding e_1, e_2 gives a basis.

Try adding these two.

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4. $L(ax^3+bx^2+cx+d) = ax^3+bx^2+cx+d$
$$-3ax^3-2bx^2-cx$$
$$= -2ax^3-bx^2+d$$

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$\mathrm{Nul}(L) = \{p \mid a=b=d=0\} = \mathrm{Span}\{x\}$
$\mathrm{Im}(L) = \mathrm{Span}\{x^3, x^2, 1\}$

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5. $L(ax^3+bx^2+cx+d) = ax^3+bx^2+cx+d$
$$-a \quad -b \quad -c \quad -d$$
$$= a(x^3-1)+b(x^2-1)+c(x-1)$$

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$\mathrm{Nul}(L) = \{p \mid a=b=c=0\} = \mathrm{Span}\{1\}$
$\mathrm{Im}(L) = \mathrm{Span}\{x^3-1, x^2-1, x-1\}$.

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6. (a) $e_1 \mapsto \begin{bmatrix}1\\2\end{bmatrix}$ so $P_{\mathcal{B}_1} = \begin{bmatrix}1 & 2\\2 & 1\end{bmatrix}$
$e_2 \mapsto \begin{bmatrix}2\\1\end{bmatrix}$

5 ea

(b) $[\cdot]_{\mathcal{B}_2}^{-1} = P_{\mathcal{B}_2} = \begin{bmatrix}1 & 1\\-1 & 1\end{bmatrix}$ so we compute the inverse:
$$\left[\begin{array}{cc|cc}1 & 1 & 1 & 0\\-1 & 1 & 0 & 1\end{array}\right] \xrightarrow{+R_1} \left[\begin{array}{cc|cc}1 & 1 & 1 & 0\\0 & 2 & 1 & 1\end{array}\right] \xrightarrow{*1/2} \left[\begin{array}{cc|cc}1 & 1 & 1 & 0\\0 & 1 & 1/2 & 1/2\end{array}\right]$$

$$\xrightarrow{-R_2} \left[\begin{array}{cc|cc}1 & 0 & 1/2 & -1/2\\0 & 1 & 1/2 & 1/2\end{array}\right] \text{ so } [\;]_{\mathcal{B}_2} = \begin{bmatrix}1/2 & -1/2\\1/2 & 1/2\end{bmatrix}$$

(c) $P_{\mathcal{B}_2 \leftarrow \mathcal{B}_1} = [\cdot]_{\mathcal{B}_2} \cdot P_{\mathcal{B}_1} = \begin{bmatrix}1/2 & -1/2\\1/2 & 1/2\end{bmatrix}\begin{bmatrix}1 & 2\\2 & 1\end{bmatrix} = \begin{bmatrix}-1/2 & 1/2\\3/2 & 3/2\end{bmatrix}$

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7. $P_3 \xrightarrow{L} \mathbb{R}^2$ by $L(p) = \begin{bmatrix} p(1) \\ p(-1) \end{bmatrix}$ want $Nul(L)$.

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or $L(ax^3+bx^2+cx+d) = \begin{bmatrix} a+b+c+d \\ -a+b-c+d \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ -1 & 1 & -1 & 1 \end{bmatrix} \xrightarrow[+R_2]{} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 2 & 0 & 2 \end{bmatrix} \xrightarrow[*1/2]{} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix} \xrightarrow{-R_2} \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

$a+c=0$
$b+d=0$

$$\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = c \begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \end{bmatrix} + d \begin{bmatrix} 0 \\ -1 \\ 0 \\ 1 \end{bmatrix}$$

$Nul(L) = \text{Span}\{x^2-1, x^3-x\}$.

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8.

5 ea

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 2 & 2 \\ 1 & 1 & 0 \end{bmatrix} \xrightarrow[-R_1]{\substack{-R_1 \\ -2R_2}} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow[-R_2]{} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ & 0 & \end{bmatrix} \xrightarrow{-R_2} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 1 \\ & 0 & \end{bmatrix}$$

(a) $x_1 - x_3 = 0$
$x_2 + x_3 = 0$

$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = x_3 \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$ $Nul(A)$ basis $\left\{ \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} \right\}$

dim = 1

(b) $Col(A)$ basis = Columns 1 & 2 $\left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 2 \\ 2 \\ 1 \end{bmatrix} \right\}$ dim = 2

(c) $Row(A)$ basis = row reduced rows 1 & 2 = $\left\{ \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \right\}$

dim = 2 = rank (A)