Math 2250, Fall 2015, Test 3 **November 23, 2015** **R. Bruner**

Each problem is worth 10 points unless otherwise indicated.

1. (20 points) Decide whether or not the following are vector spaces. Indicate why they are or why they are not.

   (a) $\left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \middle| \ x - 2y + 3z = 0 \right\}$

   (b) $\left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \middle| \ x - 2y + 3z = 1 \right\}$

   (c) $\left\{ \begin{bmatrix} x \\ x - y \\ x + 2y \end{bmatrix} \middle| \ x, y \in \mathbf{R} \right\}$

   (d) $\left\{ \begin{bmatrix} x \\ x - y \\ x + 2 \end{bmatrix} \middle| \ x, y \in \mathbf{R} \right\}$

   (e) $\left\{ \begin{bmatrix} x \\ y \end{bmatrix} \middle| \ x, y \in \mathbf{R}, x \geq 0 \right\}$

   (f) $\left\{ \begin{bmatrix} x \\ x - y \\ x + 2y \end{bmatrix} \middle| \ x, y \text{ integer} \right\}$

   (g) P_3, polynomials of degree at most 3.

   (h) $\{p(x) \in P_3 \mid \deg(p(x)) = 3\}$

   (i) $\{p(x) \in P_3 \mid p(1) = 0\}$

   (j) $\{p(x) \in P_3 \mid p(x) = xp'(x)\}$

2. Find a subset of $\left\{ \begin{bmatrix} 1 \\ 1 \\ 2 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 2 \\ 3 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \right\}$ which forms a basis for the subspace spanned by these vectors.

3. Add vectors to the set $\left\{ \begin{bmatrix} 1 \\ 2 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \right\}$ to form a basis of $\mathbf{R}^4$ which includes these two vectors.

———— Continue on reverse ————

4. Let $L : P_3 \longrightarrow P_3$ be $L(p) = p - xp'$. Find bases for $\mathrm{Nul}(L)$ and $\mathrm{Im}(L)$.

5. Let $L : P_3 \longrightarrow P_3$ be $L(p) = p - p(1)$. Find bases for $\mathrm{Nul}(L)$ and $\mathrm{Im}(L)$.

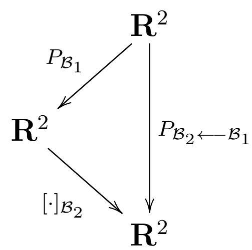
6. (15 points) Let

$$\mathcal{B}_1 = \left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \end{bmatrix} \right\} \qquad \text{and} \qquad \mathcal{B}_2 = \left\{ \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}.$$

(a) Write a 2×2 matrix for the linear transformation $P_{\mathcal{B}_1}$ which sends a coefficient matrix $[v]_{\mathcal{B}_1} = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$ to $v = c_1 \begin{bmatrix} 1 \\ 2 \end{bmatrix} + c_2 \begin{bmatrix} 2 \\ 1 \end{bmatrix}$.

(b) Write a 2×2 matrix for the linear transformation $[\cdot]_{\mathcal{B}_2}$ which computes the coordinates of a vector $v \in \mathbf{R}^2$ with respect to the basis $\mathcal{B}_2$.

(c) Write a 2×2 matrix for the linear transformation $P_{\mathcal{B}_2 \longleftarrow \mathcal{B}_1}$ which changes cordinates from basis $\mathcal{B}_1$ to basis $\mathcal{B}_2$: $P_{\mathcal{B}_2 \longleftarrow \mathcal{B}_1}[v]_{\mathcal{B}_1} = [v]_{\mathcal{B}_2}$.



7. Find a basis for the set of all polynomials of degree at most three whose values are zero at both 1 and -1.

8. (15 points) Let

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 2 & 2 \\ 1 & 1 & 0 \end{bmatrix}$$

(a) Find the dimension of and a basis for the null space of A.

(b) Find the dimension of and a basis for the column space of A.

(c) Find the dimension of and a basis for the row space of A.

——— The End ———