

Test 2 M 2250 F 15

1.  $\left[ \begin{array}{cc|c} 1 & 1 & 11 \\ -3 & 2 & 2 \end{array} \right] \xrightarrow{+3R_1} \left[ \begin{array}{cc|c} 1 & 1 & 11 \\ 0 & 5 & 35 \end{array} \right] \xrightarrow{*1/5} \left[ \begin{array}{cc|c} 1 & 1 & 11 \\ 0 & 1 & 7 \end{array} \right]$

$\xrightarrow{-R_2} \left[ \begin{array}{cc|c} 1 & 0 & 4 \\ 0 & 1 & 7 \end{array} \right] \quad \begin{bmatrix} 11 \\ 2 \end{bmatrix} = \begin{bmatrix} 4 \\ 7 \end{bmatrix}$

Check:  $4 \begin{bmatrix} 1 \\ -3 \end{bmatrix} + 7 \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 4+7 \\ -12+14 \end{bmatrix} = \begin{bmatrix} 11 \\ 2 \end{bmatrix} \checkmark$

10:45

2. det of the row reduced matrix is  $4 \cdot 3 \cdot 6$ ; Undoing the effects of the row operations:

$4 \cdot 3 \cdot 6 \mapsto 4 \cdot 3 \cdot 6 \mapsto 4 \cdot 4 \cdot 3 \cdot 6 \mapsto 4^2 \cdot 3 \cdot 6 \mapsto -4^2 \cdot 3 \cdot 6$

= -288

Next 4: pivots in columns 1, 2, 4.

10:46

3. Basis (Col(A)) =  $\left( \begin{bmatrix} 1 \\ 0 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \\ 4 \\ 3 \end{bmatrix} \right)$

10:47

4.  $\dim(\text{Col}(A)) = 3$  and  $\dim(\text{Nul}(A)) = 6 - 3 = 3$

5.  $x_1 + 2x_3 + x_5 - 2x_6 = 0 \quad x_1 = -2x_3 - x_5 + 2x_6$   
 $x_2 - 3x_3 - x_6 = 0 \quad x_2 = 3x_3 + x_6$   
 $x_4 + x_5 = 0 \quad x_4 = -x_5$   
 $x_3 = x_3$   
 $x_5 = x_5$   
 $x_6 = x_6$

$N = \left( \begin{bmatrix} -2 \\ 3 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 0 \\ -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right)$

6.  $\begin{bmatrix} -2 & -1 & 2 \\ 3 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

10:50

7.

$$(a) \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$R_2 \leftrightarrow R_3$$

$$(b) \begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_1 + 5R_3 \rightarrow R_1$$

$$(c) \begin{bmatrix} 4 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$4R_1 \rightarrow R_1$$

10:52

8.

$$A \begin{bmatrix} 1 & 2 & 4 & | & 1 & 0 & 0 \\ 1 & 3 & 6 & | & 0 & 1 & 0 \\ 1 & 5 & 9 & | & 0 & 0 & 1 \end{bmatrix} \xrightarrow{\substack{-R_1 \\ -R_1}} \begin{bmatrix} 1 & 2 & 4 & | & 1 & 0 & 0 \\ 0 & 1 & 2 & | & -1 & 1 & 0 \\ 0 & 3 & 5 & | & -1 & 0 & 1 \end{bmatrix} \xrightarrow{\substack{-2R_2 \\ -3R_2}}$$

$$\begin{bmatrix} 1 & 0 & 0 & | & 3 & -2 & 0 \\ 0 & 1 & 2 & | & -1 & 1 & 0 \\ 0 & 0 & -1 & | & 2 & -3 & 1 \end{bmatrix} \xrightarrow{\substack{+2R_3 \\ * -1}} \begin{bmatrix} 1 & 0 & 0 & | & 3 & -2 & 0 \\ 0 & 1 & 0 & | & 3 & -5 & 2 \\ 0 & 0 & 1 & | & -2 & 3 & -1 \end{bmatrix}$$

Det 3  
Inv 7

$$\text{so } A^{-1} = \begin{bmatrix} 3 & -2 & 0 \\ 3 & -5 & 2 \\ -2 & 3 & -1 \end{bmatrix}$$

Det was unaffected by the first two steps in row reduction, so equals det of the 3<sup>rd</sup>,  $\text{Det}(A) = 1 \cdot 1 \cdot (-1) = -1$ .

9.

10:57

$$\begin{bmatrix} -1 & 1 & -2 \\ 2 & 1 & 20 \\ 3 & -2 & 11 \end{bmatrix} \xrightarrow{\substack{+2R_1 \\ +3R_1}} \begin{bmatrix} -1 & 1 & -2 \\ 0 & 3 & 16 \\ 0 & 1 & 5 \end{bmatrix} \xrightarrow{-3R_3} \begin{bmatrix} -1 & 1 & -2 \\ 0 & 3 & 16 \\ 0 & 0 & 1 \end{bmatrix} \leftarrow \text{inconsistent}$$

No it is not in  $\text{Col}(A)$ .

11:00

10. (a)  $\begin{bmatrix} 1 \\ y \\ z \end{bmatrix} + \begin{bmatrix} 1 \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} 2 \\ y+y' \\ z+z' \end{bmatrix}$  is not there NOT

(b)  $x_1 = y_1 + z_1$  and  $x_2 = y_2 + z_2$  IS  
 $x_2 = y_2 + z_2$   
 (Nul  $(\begin{bmatrix} 1 & -1 & -1 \end{bmatrix})$ )

(c) Nul  $(\begin{bmatrix} 1 & -1 & -1 \\ 1 & 1 & -1 \end{bmatrix})$  IS

(d) This is just  $\mathbb{R}^2$  IS  
 written in a funny way.

(e) Col  $(\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix})$  IS

since  $\begin{bmatrix} a \\ b \\ b-a \end{bmatrix} = a \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} + b \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$ .

11:03

11. (a) Too small No  
 (b) Lin. Dep. No  
 (c) Lin. Dep. No  
 (d) Basis YES  
 (e) Too Large. No