Math 2250, Fall 2015, Test 2 **October 30, 2015** **R. Bruner**

1. Find the coordinates of the vector $\begin{bmatrix} 11 \\ 2 \end{bmatrix}$ with respect to the basis $\begin{bmatrix} 1 \\ -3 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \end{bmatrix}$.

2. The row operations

$$R_1 \longleftrightarrow R_3, \qquad R_2 + 5R_1 \longrightarrow R_2, \qquad \frac{1}{4}R_3 \longrightarrow R_3, \qquad \text{and} \qquad R_3 + R_2 \longrightarrow R_3$$

were used to reduce the matrix A to

$$\begin{bmatrix} 4 & 1 & 7 \\ 0 & 3 & 3 \\ 0 & 0 & 6 \end{bmatrix}$$

What is $\det(A)$?

The next four problems concern the matrix A, which has reduced row echelon form R:

$$A = \begin{bmatrix} 1 & 0 & 2 & -1 & 0 & -2 \\ 0 & 1 & -3 & 2 & 2 & -1 \\ 1 & 2 & -4 & 4 & 5 & -4 \\ 2 & 2 & -2 & 3 & 5 & -6 \end{bmatrix} \qquad \longrightarrow \qquad R = \begin{bmatrix} 1 & 0 & 2 & 0 & 1 & -2 \\ 0 & 1 & -3 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

3. Find a basis $\mathcal{C}$ for the column space $\text{col}(A)$.

4. What are the dimensions $\dim(\text{col}(A))$ and $\dim(\text{nul}(A))$?

5. Find a basis $\mathcal{N}$ for the null space $\text{nul}(A)$.

6. Find a matrix N with $\text{col}(N) = \text{nul}(A)$.

7. Write 3×3 elementary matrices which correspond to the following row operations

$$(a)\ R_2 \longleftrightarrow R_3 \qquad (b)\ R_1 + 5R_3 \longrightarrow R_1 \qquad (c)\ 4R_1 \longrightarrow R_1$$

8. Compute the determinant and inverse of

$$\begin{bmatrix} 1 & 2 & 4 \\ 1 & 3 & 6 \\ 1 & 5 & 9 \end{bmatrix}$$

9. (5) Determine whether or not $\begin{bmatrix} -2 \\ 20 \\ 11 \end{bmatrix}$ is in $\text{col}(A)$ if $A = \begin{bmatrix} -1 & 1 \\ 2 & 1 \\ 3 & -2 \end{bmatrix}$.

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10. For each of the following, determine whether it is a subspace of $\mathbf{R}^2$ or not.

(a) The vectors $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ such that $x = 1$.

(b) The vectors $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ such that $x = y + z$.

(c) The vectors $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ such that $x = y + z$ and $x + y = z$.

(d) The vectors $\begin{bmatrix} a+1 \\ b+1 \end{bmatrix}$ for all real numbers a and b.

(e) The vectors $\begin{bmatrix} a \\ b \\ b-a \end{bmatrix}$ for all real numbers a and b.

11. (5) Which of these are bases for R^3?

(a) $\left\{ \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 3 \\ 3 \\ 0 \end{bmatrix} \right\}$

(b) $\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$

(c) $\left\{ \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 3 \\ 0 \end{bmatrix} \right\}$

(d) $\left\{ \begin{bmatrix} 0 \\ 0 \\ 7 \end{bmatrix}, \begin{bmatrix} 0 \\ 3 \\ 0 \end{bmatrix}, \begin{bmatrix} 9 \\ 0 \\ 0 \end{bmatrix} \right\}$

(e) $\left\{ \begin{bmatrix} 1 \\ 2 \\ 7 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ 4 \end{bmatrix}, \begin{bmatrix} 4 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 6 \\ 2 \\ 5 \end{bmatrix} \right\}$

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