M2250 F15 TEST 1 SOL.

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1. (a) ⑤

$$-x_1 + 2x_2 + x_3 = 4$$
$$2x_1 + x_2 + 8x_3 = 7$$
$$3x_1 - x_2 + 7x_3 = 3$$

(b) ⑤

$$\left[\begin{array}{ccc|c} -1 & 2 & 1 & 4 \\ 2 & 1 & 8 & 7 \\ 3 & -1 & 7 & 3 \end{array}\right]$$

(c) ⑩

$$\begin{array}{c} \xrightarrow{*-1} \\ \xrightarrow{+2R_1} \\ \xrightarrow{+3R_1} \end{array} \left[\begin{array}{ccc|c} 1 & -2 & -1 & -4 \\ 0 & 5 & 10 & 15 \\ 0 & 5 & 10 & 15 \end{array}\right] \begin{array}{c} \xrightarrow{*1/5} \\ \xrightarrow{-R_2} \end{array} \left[\begin{array}{ccc|c} 1 & -2 & -1 & -4 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right] \xrightarrow{+2R_2}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 3 & 2 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right] \qquad \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -3 \\ -2 \\ 1 \end{bmatrix}$$

(d) ⑤ $\begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$

(e) ⑤ $x_3 \begin{bmatrix} -3 \\ -2 \\ 1 \end{bmatrix}$

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2. (a) ⑤

$$T\begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} = 3\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} + 2\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} + \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 3+2+2 \\ 3+4+1 \\ 6+2+1 \end{bmatrix} = \begin{bmatrix} 7 \\ 8 \\ 9 \end{bmatrix}$$

(b) ⑤ (c) ⑤

$$\begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 1 \\ 2 & 1 & 1 \end{bmatrix} \begin{array}{c} \xrightarrow{-R_1} \\ \xrightarrow{-2R_1} \end{array} \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & -1 \\ 0 & -1 & -3 \end{bmatrix} \xrightarrow[+R_2]{*} \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & -4 \end{bmatrix}$$

YES

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3. (a) 10 Solution is $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = x_3 \begin{bmatrix} -2 \\ 3 \\ 1 \\ 0 \end{bmatrix}$ or $-2\vec{v}_1 + 3\vec{v}_2 + \vec{v}_3 = 0$

(b) 5 v_3 is dependent on v_1, v_2 so elim it. others have pivots so $\{v_1, v_2, v_4\}$

(c) 5 $v_3 = 2v_1 - 3v_2$

(d) 5 No. can't solve $Av = b$ for every b because there is no pivot in row 4.

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4. 10 $\left[\begin{array}{cc|c} 1 & 2 & 3 \\ 2 & 1 & h \\ 1 & 1 & -2 \end{array}\right] \xrightarrow[-R_1]{-2R_1} \left[\begin{array}{cc|c} 1 & 2 & 3 \\ 0 & -3 & h-6 \\ 0 & -1 & -5 \end{array}\right] \xrightarrow{-3R_3} \left[\begin{array}{cc|c} 1 & 2 & 3 \\ 0 & 0 & h-6+15 \\ 0 & -1 & -5 \end{array}\right]$

$h + 9 = 0$ OR $\boxed{h = -9}$

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5. 10 $x_1(u - v) + x_2(v - w) + x_3(u - w) = 0$ gives

$(x_1 + x_3)u + (-x_1 + x_2)v + (-x_2 - x_3)w = 0$.

Since $\{u, v, w\}$ is lin. indep. this implies
$$\begin{aligned} x_1 + x_3 &= 0 \\ -x_1 + x_2 &= 0 \\ -x_2 - x_3 &= 0 \end{aligned}$$

Now $\begin{bmatrix} 1 & 0 & 1 \\ -1 & 1 & 0 \\ 0 & -1 & -1 \end{bmatrix} \xrightarrow{+R_1} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & -1 & -1 \end{bmatrix} \xrightarrow{+R_2} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

so the system has non-zero solutions. Not lin. indep.

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6. 10 $u + 2v = 2(2u + v) - 3u$ so $T(u + 2v) = 2T(2u + v) - 3T(u)$

$= \begin{bmatrix} 2 \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 3 \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$

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