Math 2250, Fall 2015, Test 1

R. Bruner

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1. Write the vector equation

$$x_1 \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix} + x_2 \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix} + x_3 \begin{bmatrix} 1 \\ 8 \\ 7 \end{bmatrix} = \begin{bmatrix} 4 \\ 7 \\ 3 \end{bmatrix}$$

   (a) (5 pts) as a system of linear equations

   (b) (5 pts) in matrix form.

   (c) (10 pts) Find the set of all solutions $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$.

   (d) (5 pts) Give two particular solutions to the system.

   (e) (5 pts) Find the solution set to the associated homogeneous system.

2. (5 pts each) Let $T : \mathbf{R}^3 \longrightarrow \mathbf{R}^3$ be the linear transformation satisfying

$$T \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}, \qquad T \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \qquad T \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}.$$

   (a) Find $T \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$.

   (b) Write the matrix form of T.

   (c) Can the equation $T(\mathbf{v}) = \mathbf{b}$ be solved for every $\mathbf{b}$?

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3. Let

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \end{bmatrix}, \qquad \mathbf{v}_2 = \begin{bmatrix} -1 \\ -2 \\ 3 \\ 1 \end{bmatrix}, \qquad \mathbf{v}_3 = \begin{bmatrix} 5 \\ 8 \\ -11 \\ -5 \end{bmatrix}, \qquad \mathbf{v}_4 = \begin{bmatrix} -5 \\ 1 \\ 1 \\ 1 \end{bmatrix}.$$

The reduced row echelon form of $A = [\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4]$ is $\begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & -3 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$.

(a) (10 pts) Find a nontrivial linear combination of $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$, and $\mathbf{v}_4$.

(b) (5 pts) Find a linearly independent subset of these 4 vectors which is as large as possible.

(c) (5 pts) Write the remaining vector(s) as a linear combination of the ones you identified as forming a linearly independent set.

(d) (5 pts) Does this set of four vectors span $\mathbf{R}^4$?

4. (10 pts) Suppose that $\begin{bmatrix} 3 \\ h \\ -2 \end{bmatrix}$ is in the span of $\left\{ \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} \right\}$. What must h be?

5. (10 pts) Suppose that $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is a linearly independent set. Is $\{\mathbf{u} - \mathbf{v}, \mathbf{v} - \mathbf{w}, \mathbf{u} - \mathbf{w}\}$ linearly independent or not? Why? (Hint: it may help to write out the relevant equations carefully and turn this into a system of equations to be solved.)

6. (10 pts) If T is a linear transformation with $T(2\mathbf{u} + \mathbf{v}) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $T(\mathbf{u}) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, compute $T(\mathbf{u} + 2\mathbf{v})$.

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