

Name: _____

Quiz 7

Math 2250, Fall 2015

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R. Bruner

1. Find the coordinates of the vector $\begin{bmatrix} -11 \\ -3 \end{bmatrix}$ with respect to the basis $\left\{ \begin{bmatrix} 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ 4 \end{bmatrix} \right\}$.
2. The row operations $R_1 \leftrightarrow R_2$, $R_2 - 3R_1 \rightarrow R_2$, $2R_3 \rightarrow R_3$, and $R_3 + 4R_2 \rightarrow R_3$ were used to reduce the matrix A to

$$\begin{bmatrix} 1 & 1 & 7 \\ 0 & 2 & 3 \\ 0 & 0 & 6 \end{bmatrix}$$

What is $\det(A)$?

————— Solutions —————

1. We wish to solve

$$x_1 \begin{bmatrix} 1 \\ 3 \end{bmatrix} + x_2 \begin{bmatrix} 3 \\ 4 \end{bmatrix} = \begin{bmatrix} -11 \\ -3 \end{bmatrix}$$

or

$$\begin{bmatrix} x_1 + 3x_2 \\ 3x_1 + 4x_2 \end{bmatrix} = \begin{bmatrix} -11 \\ -3 \end{bmatrix}$$

We do this by row reduction:

$$\left[\begin{array}{cc|c} 1 & 3 & -11 \\ 3 & 4 & -3 \end{array} \right] \rightarrow \left[\begin{array}{cc|c} 1 & 0 & 7 \\ 0 & 1 & -6 \end{array} \right]$$

Hence, the coordinates of $\begin{bmatrix} -11 \\ -3 \end{bmatrix}$ with respect to the basis $\left(\begin{bmatrix} 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ 4 \end{bmatrix} \right)$ are $\begin{bmatrix} 7 \\ -6 \end{bmatrix}$.

2. The row reduced matrix has determinant $1 \cdot 2 \cdot 6 = 12$. Reversing the effects of the row operations we have

$$12 \mapsto 12 \mapsto \frac{1}{2} \cdot 12 = 6 \mapsto 6 \mapsto \det(A) = -6$$