Name: _____

Quiz 12
Math 2250, Fall 2015 **December 14, 2015** **R. Bruner**

1. Using inner products rather than row reduction, write $\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$ as a linear combination of the vectors in the orthogonal basis $\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ -1 \end{bmatrix} \right\}$.

2. Find the point of W closest to $\begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$ if W is the span of the orthogonal set $\left\{ \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix} \right\}$.

(1.) $u_i \cdot u_i = 2$ for all 4 basis vectors

$y \cdot u_1 = 1+2 = 3$ $y \cdot u_2 = 1-2 = -1$ $y \cdot u_3 = 3+4 = 7$ $y \cdot u_4 = 3-4 = -1$

So $\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} = \frac{3}{2}\begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} - \frac{1}{2}\begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix} + \frac{7}{2}\begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} - \frac{1}{2}\begin{bmatrix} 0 \\ 0 \\ 1 \\ -1 \end{bmatrix}$

(2.) $y \cdot u_1 = 1+4+2 = 7$ $y \cdot u_2 = -1+2-2 = -1$

$u_1 \cdot u_1 = 1+4+1 = 6$ $u_2 \cdot u_2 = 1+1+1 = 3$

So

Closest point $= \text{proj}_W y = \frac{7}{6}\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} - \frac{1}{3}\begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 7/6 + 1/3 \\ 14/6 - 1/3 \\ 7/6 + 1/3 \end{bmatrix} = \begin{bmatrix} 3/2 \\ 2 \\ 3/2 \end{bmatrix}$