Name: ______________________________

Quiz 11
Math 2250, Fall 2015 **December 4, 2015** **R. Bruner**

Find the eigenvalues and corresponding eigenvectors for

$$A = \begin{bmatrix} 8 & 2 \\ 3 & 3 \end{bmatrix}.$$

Extra credit: Find a diagonal matrix D and an invertible matrix P such that

$$A = PDP^{-1}.$$

$$\det(A-\lambda I) = \det\begin{bmatrix} 8-\lambda & 2 \\ 3 & 3-\lambda \end{bmatrix} = (8-\lambda)(3-\lambda) - 6$$
$$= 24 - 11\lambda + \lambda^2 - 6$$
$$= \lambda^2 - 11\lambda + 18$$
$$= (\lambda-2)(\lambda-9)$$

The eigenvalues are therefore $\lambda = 2$ and $\lambda = 9$.

For $\lambda_1 = 2$: $A - 2I = \begin{bmatrix} 6 & 2 \\ 3 & 1 \end{bmatrix} \xrightarrow{-2R_2} \begin{bmatrix} 0 & 0 \\ 3 & 1 \end{bmatrix}$ Nulspace requires

$3x_1 + x_2 = 0$ or $x_2 = -3x_1$ so $v_1 = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$ is an eigenvector.

For $\lambda_2 = 9$: $A - 9I = \begin{bmatrix} -1 & 2 \\ 3 & -6 \end{bmatrix} \xrightarrow[+3R_1]{} \begin{bmatrix} -1 & 2 \\ 0 & 0 \end{bmatrix}$ Nulspace requires that

$-x_1 + 2x_2 = 0$ or $x_1 = 2x_2$ so $v_2 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ is an eigenvector.

EC:

$$\begin{tikzcd} \mathbb{R}^2 \arrow[r, "A"] \arrow[d, "P^{-1}"'] & \mathbb{R}^2 \\ \mathbb{R}^2 \arrow[r] & \mathbb{R}^2 \arrow[u, "P"'] \end{tikzcd}$$

$D = \begin{bmatrix} 2 & \\ & 9 \end{bmatrix}$

$$A = \begin{bmatrix} 8 & 2 \\ 3 & 3 \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 2 \\ -3 & 1 \end{bmatrix}}_{P} \underbrace{\begin{bmatrix} 2 & 0 \\ 0 & 9 \end{bmatrix}}_{D} \underbrace{\begin{bmatrix} 1 & 2 \\ -3 & 1 \end{bmatrix}^{-1}}_{P^{-1}}$$

$$\begin{tikzcd} v_1 \arrow[r, mapsto] & 2v_1 & & v_2 \arrow[r, mapsto] & 9v_2 \\ e_1 \arrow[u] \arrow[r, mapsto] & 2e_1 \arrow[u] & P & e_2 \arrow[u] \arrow[r, mapsto] & 9e_2 \arrow[u] \end{tikzcd}$$

shows that P must send e_i to v_i.