

Name: _____

Quiz 9a = 10

Math 2250, Fall 2015

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1. Let $L : P_2 \rightarrow P_2$ be $L(p(x)) = p(x) - p(0)$. Compute $\text{Nul}(L)$ and $\text{Im}(L)$. (Write them as Span of a set of polynomials. You may assume that L is linear.)
(HINT: Write $p(x) = ax^2 + bx + c$ to do the calculation.)
2. Let $E : P_2 \rightarrow \mathbf{R}$ be the error in the trapezoidal approximation to $\int_{-1}^1 p(t) dt$:

$$E(p(x)) = \int_{-1}^1 p(t) dt - (p(1) + p(-1))$$

Compute $\text{Nul}(E)$. (Extra credit: Do this for P_3 rather than P_2 .)

① $L[ax^2 + bx + c] = ax^2 + bx + c - c = ax^2 + bx$

$$\text{Nul}(A) = \text{Span}\{1\}$$

$$\text{Col}(A) = \text{Span}\{x^2, x\}$$

② $E(ax^2 + bx + c) = \left(\int_{-1}^1 at^2 + bt + c \right) - (a+b+c + a-b+c)$

$$= \left. \frac{1}{3}at^3 + \frac{1}{2}bt^2 + ct \right|_{-1}^1 - (2a+2c)$$

$$= \frac{2}{3}a + 2c - 2a - 2c = -\frac{4}{3}a = 0 \text{ if } p \in \text{Nul}(E)$$

so $\text{Nul}(E) = \text{Span}\{x, 1\}$.