Name: ________________________________

Worksheet and Quiz 3
Math 2250, Fall 2015 **September 23, 2015** **R. Bruner**

1. Let T be a linear transformation $\mathbf{R}^2 \longrightarrow \mathbf{R}^3$. such that

$$T\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 7 \\ 1 \\ 3 \end{bmatrix} \qquad \text{and} \qquad T\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$$

   (a) Find $T\begin{bmatrix} 1 \\ 5 \end{bmatrix}$.

   (b) Find the matrix representation of T.

2. Let T be a linear transformation $\mathbf{R}^2 \longrightarrow \mathbf{R}^3$ such that

$$T\begin{bmatrix}1\\1\end{bmatrix} = \begin{bmatrix}7\\1\\3\end{bmatrix} \qquad \text{and} \qquad T\begin{bmatrix}1\\2\end{bmatrix} = \begin{bmatrix}1\\1\\-1\end{bmatrix}.$$

(a) Write $\begin{bmatrix}0\\1\end{bmatrix}$ as a linear combination of $\begin{bmatrix}1\\1\end{bmatrix}$ and $\begin{bmatrix}1\\2\end{bmatrix}$.

(b) Find $T\begin{bmatrix}0\\1\end{bmatrix}$.

(c) Write $\begin{bmatrix}1\\0\end{bmatrix}$ as a linear combination of $\begin{bmatrix}1\\1\end{bmatrix}$ and $\begin{bmatrix}1\\2\end{bmatrix}$.

(d) Find $T\begin{bmatrix}1\\0\end{bmatrix}$.

(e) Find the matrix representation of T.

3. How many vectors can a linearly independent set in $\mathbf{R}^3$ have? Choose as many as are correct:

   (a) 1?

   (b) 2?

   (c) 3?

   (d) 4?

   (e) 5?

   (f) infinitely many?

4. Write the equation which expresses the fact that $\begin{bmatrix} 7 \\ 5 \end{bmatrix}$ is in the span of $\begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \end{bmatrix}$, and $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$.

   You may use indeterminate coefficients x_i. You do not need to find the values of the coefficients.

5. Find all ways of writing $\begin{bmatrix} 7 \\ 5 \end{bmatrix}$ as a linear combination of $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$, $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$, and $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$.

6. Let $\mathbf{a}_1 = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$, $\mathbf{a}_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ and $\mathbf{a}_3 = \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}$ and $\mathbf{a}_4 = \begin{bmatrix} 4 \\ 0 \\ 4 \end{bmatrix}$.

(a) Show that $\{\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3, \mathbf{a}_4\}$ is linearly dependent by finding a nontrivial linear combination of the vectors which is 0.

(b) Find a linearly independent subset of them which contains as many vectors as possible.

(c) Express the remaining vector (or vectors) as a linear combination of your linearly independent subset.

7. If $\mathbf{a}_1$, $\mathbf{a}_2$, and $\mathbf{a}_3$, are linearly independent and $\mathbf{a}_4$ is not in their span, show that $\{\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3, \mathbf{a}_4\}$ is linearly independent.

8. Suppose that $\mathbf{a}_1 - 2\mathbf{a}_2 + \mathbf{a}_4 = 0$ and $\mathbf{a}_1 + \mathbf{a}_3 + \mathbf{a}_5 = 0$. Show that $\{\mathbf{a}_2, \mathbf{a}_3, \mathbf{a}_4, \mathbf{a}_5\}$ is linearly dependent.