Solutions to some additional practice
Math 2250, Fall 2015 **R. Bruner**

1. $L : P_2 \longrightarrow P_2$ by $L(p) = p - p(1)$.

   We have

$$\begin{aligned} L(ax^2 + bx + c) = & \quad ax^2 + bx + c - a - b - c \\ = & \quad a(x^2 - 1) + b(x - 1) \end{aligned}$$

   so $\mathrm{Nul}(L) = \{p \mid a = b = 0\} = \mathrm{Span}\{1\}$ and $\mathrm{Im}(L) = \mathrm{Span}\{x^2 - 1, x - 1\}$.

2. $L : P_2 \longrightarrow \mathbf{R}$ by $L(p) = p(1)$.

   $L(ax^2 + bx + c) = a + b + c$, so $\mathrm{Nul}(L) = \{p \mid a = -b - c\} = \mathrm{Span}\{x^2 - 1, x - 1\}$ and $\mathrm{Im}(L) = \mathbf{R} = \mathrm{Span}\{1\}$.

3. $L : P_2 \longrightarrow \mathbf{R}$ by $L(p) = p(1) - p(0)$.

   $L(ax^2 + bx + c) = a + b$, so $\mathrm{Nul}(L) = \{p \mid a = -b\} = \mathrm{Span}\{x^2 - x, 1\}$ and $\mathrm{Im}(L) = \mathbf{R} = \mathrm{Span}\{1\}$.

4.

$$E(p(x)) = \int_{-1}^{1} p(t)dt - 2p(0)$$

   $E(ax^2 + bx + c) = \frac{2}{3}a + 2c - (2c) = \frac{2}{3}a$, so $\mathrm{Nul}(E) = \{p \mid a = 0\} = \mathrm{Span}\{x, 1\}$.

   $E(ax^3 + bx^2 + cx + d) = \frac{2}{3}b + 2d - (2d) = \frac{2}{3}b$, so $\mathrm{Nul}(E) = \{p \mid b = 0\} = \mathrm{Span}\{x^3, x, 1\}$.