$$\left[\begin{array}{ccc|c} 1 & -4 & 7 & g \\ 0 & 3 & -5 & h \\ -2 & 5 & -9 & k \end{array}\right] \xrightarrow[+2R_1]{} \left[\begin{array}{ccc|c} 1 & -4 & 7 & g \\ 0 & 3 & -5 & h \\ 0 & -3 & 5 & k+2g \end{array}\right]$$

$$\xrightarrow{+R_2} \left[\begin{array}{ccc|c} 1 & -4 & 7 & g \\ 0 & 3 & -5 & h \\ \hline 0 & 0 & 0 & k+2g+h \end{array}\right]$$

The system has a solution if

$$k+2g+h=0$$

Otherwise it has no solution.

$$\left[\begin{array}{ccc|c} 1 & -4 & 7 & g \\ 0 & 1 & -5/3 & h/3 \end{array}\right] \xrightarrow{+4R_2} \left[\begin{array}{ccc|c} 1 & 0 & 1/3 & g+\frac{4}{3}h \\ 0 & 1 & -5/3 & h/3 \end{array}\right]$$

Just for fun

Solutions:

$$x_1 = g+\tfrac{4}{3}h - \tfrac{1}{3}x_3$$
$$x_2 = h/3 \quad +\tfrac{5}{3}x_3$$
$$x_3 = x_3$$

OR

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} g+\frac{4}{3}h \\ h/3 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -1/3 \\ 5/3 \\ 1 \end{bmatrix}$$

Then $x_1-4x_2+7x_3 = g+\frac{4}{3}h-\frac{1}{3}x_3-\frac{4h}{3}-\frac{20}{3}x_3+\frac{21}{3}x_3 = g+\left(\frac{4}{3}-\frac{4}{3}\right)h+\left(\frac{-1}{3}-\frac{20}{3}+\frac{21}{3}\right)x_3$

$= g$ OKAY

$3x_2-5x_3 = \frac{3h}{3}+5x_3-5x_3 = h$ OKAY

$-2x_1+5x_2-9x_3 = -2g-\frac{8}{3}h+\frac{2}{3}x_3+\frac{5}{3}h+\frac{25}{3}x_3-\frac{27}{3}x_3 = -2g-h$

And this $=k$ if $2g+h+k=0$.