

1. (b) Undefined  
(g) Does not exist  
(i) Undefined

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5 2.  $2a_1 + 6a_2 + 7a_3$

5 3. To write  $\begin{bmatrix} 1 \\ 2 \end{bmatrix} = c_1 \begin{bmatrix} 4 \\ 3 \end{bmatrix} + c_2 \begin{bmatrix} 3 \\ 2 \end{bmatrix}$  we solve

$$\left[ \begin{array}{cc|c} 4 & 3 & 1 \\ 3 & 2 & 2 \end{array} \right] \xrightarrow{-R_2} \left[ \begin{array}{cc|c} 1 & 1 & -1 \\ 3 & 2 & 2 \end{array} \right] \xrightarrow{-3R_1} \left[ \begin{array}{cc|c} 1 & 1 & -1 \\ 0 & -1 & 5 \end{array} \right] \xrightarrow{\substack{+R_2 \\ \times -1}}$$

$$\left[ \begin{array}{cc|c} 1 & 0 & 4 \\ 0 & 1 & -5 \end{array} \right]$$

$$\text{So } \begin{bmatrix} 1 \\ 2 \end{bmatrix} = 4 \begin{bmatrix} 4 \\ 3 \end{bmatrix} - 5 \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

5 4.  $\dim \text{col}(A) = 3 - \dim \text{null}(A) = 1$

5 5.  $\dim(\text{Space of solutions}) = \dim \text{null } A$   
 $= 7 - 3 = 4$

$$\mathbb{R}^7 \xrightarrow{A} \mathbb{R}^6$$

$\cup$   
Null(A)

15 6. (a)  $4 \times 7 \times 10 \times 9$

(b)  $2 \begin{bmatrix} 0 & 6 & 5 \\ 3 & 7 & 8 \\ 0 & 0 & 2 \end{bmatrix} = 2(-3) \begin{vmatrix} 6 & 5 \\ 0 & 2 \end{vmatrix} = -6 \times 12$

(c) 0 (Two rows are identical)

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7. 
$$\begin{bmatrix} 1 & -2 & 1 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 1-4+3 \\ 1+2-3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \underline{\text{YES}}$$

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8. 
$$\left[ \begin{array}{cc|c} 1 & 1 & 1 \\ -2 & 1 & 2 \\ 1 & -1 & 3 \end{array} \right] \xrightarrow{\substack{+2R_1 \\ -R_1}} \left[ \begin{array}{cc|c} 1 & 1 & 1 \\ 0 & 3 & 4 \\ 0 & -2 & 2 \end{array} \right] \left\{ \begin{array}{l} \leftarrow x_2 = 4/3 \\ \leftarrow x_2 = -1 \end{array} \right.$$

last two rows  
inconsistent. NO

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9. Bases: (a), (c)

(b) 
$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ -1 & 1 & 0 \end{bmatrix}$$

(d) & (e) are the wrong size

<sup>A</sup> Think col = first + second

10.  $\text{col}(A)$  has basis  $\left\{ \begin{bmatrix} -3 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \\ 5 \end{bmatrix} \right\}$

10  
(5 ea)

Null(A):

$$x_1 = 2x_2 + x_4 - 3x_5$$

$$x_2 = x_2$$

$$x_3 = -2x_4 + 2x_5$$

$$x_4 = x_4$$

$$x_5 = x_5$$

$$\text{Basis}(\text{Null}(A)) = \left\{ \begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ -2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 0 \\ 2 \\ 0 \\ 1 \end{bmatrix} \right\}$$

11. (a) No  $\begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$  is not in (a)

(b) No. If  $x - 2y = 1$  then  $2x - 2(2y) = 2$

(c) Yes, =  $\text{Span} \left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \end{bmatrix} \right\}$

(d) Yes =  $\text{Null} \left( \begin{bmatrix} 1 & -1 & -1 \\ 1 & 2 & -6 \end{bmatrix} \right)$

(e) No constant 1 in  $1-b$  "displaces it"

$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$  is not in it. ( $b=1$  to make 2<sup>nd</sup> coord 0  
but  $b=0$  to make 3<sup>rd</sup> coord 0)

12. (a)  $\begin{bmatrix} 1/2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1/5 \\ 0 & 0 & 1/4 \end{bmatrix}$

(b)  $\left[ \begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 4 & 0 & 1 & 0 \\ 0 & 0 & 2 & 0 & 0 & 1 \end{array} \right] \xrightarrow{-2R_2} \left[ \begin{array}{ccc|ccc} 1 & 0 & -5 & 1 & -2 & 0 \\ 0 & 1 & 4 & 0 & 1 & 0 \\ 0 & 0 & 2 & 0 & 0 & 1 \end{array} \right] \xrightarrow{\begin{array}{l} -2R_3 \\ *1/2 \end{array}} \left[ \begin{array}{ccc|ccc} 1 & 0 & -5 & 1 & -2 & 0 \\ 0 & 1 & 4 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1/2 \end{array} \right] \xrightarrow{+5R_3}$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 0 & 0 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & -2 & 5/2 \\ 0 & 1 & -2 \\ 0 & 0 & 1/2 \end{bmatrix}$$

2 (c)  $(A^{-1}B)^{-1} = B^{-1}(A^{-1})^{-1} = B^{-1}A$

2 (d)  $(ABC)^{-1} = C^{-1}B^{-1}A^{-1}$

13. (c) is False

(h) is False

(i) is False

14. (a) Not a vector space:  $x^2 + x^2 = 2x^2$  is not in it, but  $x^2$  is.

(b) Vector Space ( $= \text{null} \left( \begin{bmatrix} ev_0 \\ ev_1 \\ ev_2 \end{bmatrix} \right)$ )

(c) Vector Space ( $= \text{Null}(ev_1 - 2ev_0)$ )

15. (a) Lin Trans.

(b) Lin Trans.