Math 2250, Fall 2011, Test 2 **November 7, 2011** **R. Bruner**

1. (10) Which of the following are undefined or non-existent?

(a) $\begin{bmatrix} 1 & 0 \\ 3 & 3 \end{bmatrix} + \begin{bmatrix} 2 & 5 \\ 1 & 4 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 0 \\ 3 & 3 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 1 \\ 2 & 2 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 0 \\ 3 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 2 & 2 & 0 \end{bmatrix}$

(d) $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix}$

(e) $\begin{bmatrix} 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix}$

(f) $\begin{bmatrix} 1 & 0 \\ 3 & 3 \end{bmatrix}^{-1}$

(g) $\begin{bmatrix} 1 & 0 & 2 \\ 2 & 0 & 4 \\ 3 & 0 & 6 \end{bmatrix}^{-1}$

(h) $2\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} + 3\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}^T$

(i) $\det \begin{bmatrix} 1 & 2 & 0 \\ 3 & 0 & 1 \end{bmatrix}$

(j) $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 4 & 1 \\ 0 & 1 \\ 6 & 2 \end{bmatrix} \begin{bmatrix} 6 \\ 7 \end{bmatrix}$

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2. (5) Write $\begin{bmatrix} a_1 & a_2 & a_3 \end{bmatrix} \begin{bmatrix} 2 \\ 6 \\ 7 \end{bmatrix}$ as a linear combination.

3. (5) Find the coordinates of $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ with respect to the basis $\{\begin{bmatrix} 4 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \end{bmatrix}\}$.

4. (5) If the linear transformation $T : \mathbf{R}^3 \longrightarrow \mathbf{R}^4$ has a 2-dimensional null space, what is the dimension of its column space.

5. (5) If the 6 by 7 matrix A has rank 3, what is the dimension of the space of solutions to $Ax = 0$.

6. (15) Compute

   (a) $\det \begin{bmatrix} 4 & 1 & 3 & 3 \\ 0 & 7 & 9 & 8 \\ 0 & 0 & 10 & 6 \\ 0 & 0 & 0 & 9 \end{bmatrix}$

   (b) $\det \begin{bmatrix} 2 & 0 & 0 & 0 \\ 5 & 0 & 6 & 5 \\ 4 & 3 & 7 & 8 \\ 0 & 0 & 0 & 2 \end{bmatrix}$

   (c) $\det \begin{bmatrix} 1 & 3 & 3 & 5 & 5 \\ 4 & 1 & 1 & 7 & 7 \\ 1 & 3 & 3 & 5 & 5 \\ 6 & 5 & 4 & 3 & 2 \\ 3 & 1 & 2 & 1 & 8 \end{bmatrix}$

7. (5) Determine whether or not $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ is in null(A) if

$$A = \begin{bmatrix} 1 & -2 & 1 \\ 1 & 1 & -1 \end{bmatrix}.$$

8. (5) Determine whether or not $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ is in col(A) if

$$A = \begin{bmatrix} 1 & 1 \\ -2 & 1 \\ 1 & -1 \end{bmatrix}.$$

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9. (5) Which of these are bases for R^3?

(a) $\{\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}\}$

(b) $\{\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 3 \\ 0 \end{bmatrix}\}$

(c) $\{\begin{bmatrix} 0 \\ 0 \\ 7 \end{bmatrix}, \begin{bmatrix} 0 \\ 3 \\ 0 \end{bmatrix}, \begin{bmatrix} 9 \\ 0 \\ 0 \end{bmatrix}\}$

(d) $\{\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 3 \\ 3 \\ 0 \end{bmatrix}\}$

(e) $\{\begin{bmatrix} 1 \\ 2 \\ 7 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ 4 \end{bmatrix}, \begin{bmatrix} 4 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 6 \\ 2 \\ 5 \end{bmatrix}\}$

10. (10) The matrix A and its reduced row echelon form are

$$A = \begin{bmatrix} -3 & 6 & -1 & 1 & -7 \\ 1 & -2 & 2 & 3 & 1 \\ 2 & -4 & 5 & 8 & 4 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & -2 & 0 & -1 & 3 \\ 0 & 0 & 1 & 2 & -2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Give a basis for col(A) and a basis for null(A).

11. (5) Are the following subspaces or not?

(a) $\{\begin{bmatrix} x \\ y \end{bmatrix} \mid -1 \le x \le 1\}$

(b) $\{\begin{bmatrix} x \\ y \end{bmatrix} \mid x - 2y = 1\}$

(c) $\{\begin{bmatrix} a + 2b \\ a - b \end{bmatrix} \mid a \text{ and } b \text{ real}\}$

(d) $\{\begin{bmatrix} x \\ y \\ z \end{bmatrix} \mid x = y + z \text{ and } x + 2y - 6z = 0\}$

(e) $\{\begin{bmatrix} a + b \\ 1 - b \\ b \end{bmatrix} \mid a \text{ and } b \text{ real}\}$

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12. (10) Compute

(a) $\begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 1/3 & 0 & 0 \\ 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix}^{-1}$

(b) $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 0 & 0 & 2 \end{bmatrix}^{-1}$

(c) $(A^{-1}B)^{-1}$ assuming that A and B are invertible.

(d) $(ABC)^{-1}$ assuming that A, B and C are invertible.

13. (10) Which of the following are False?

(a) If A^{-1} exists then $Ax = b$ can be solved for every b.

(b) If A^{-1} exists then $Ax = b$ has exactly one solution.

(c) If A is 3 by 4 and A has 3 pivots then A is invertible.

(d) If $Ax = b$ and $Ay = c$ then $Av = 2b - c$ is solved by $v = 2x - y$.

(e) An elementary matrix is invertible.

(f) An invertible matrix can be written as the product of elementary matrices.

(g) If A is n by n and $AC = I$ then $CA = I$.

(h) There are subspaces of R^2 which contain exactly two elements.

(i) If Ax_1 and Ax_2 are both nonzero then $x_1 - x_2$ is in null(A).

(j) If x_1 and x_2 are in col(A) then $x_1 - x_2$ is in col(A).

14. (3) Are these vector spaces or not?

(a) Polynomials of the form $x^2 + ax + b$

(b) Polynomials whose values at 0, 1 and 2 are 0.

(c) Polynomials p which satisfy $p(1) = 2p(0)$.

15. (2) Are these linear transformations or not?

(a) $T : P_2 \longrightarrow \mathbf{R}$ by $T(ax^2 + bx + c) = a + b + c$.

(b) $T : P_2 \longrightarrow P_2$ by $T(p) = p' - p$.

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