

No calculators allowed.

Find the determinants of these matrices.

$$\begin{bmatrix} 2 & 0 & 1 & 0 & 3 \\ 2 & 3 & 5 & 0 & 1 \\ 0 & 0 & 0 & 0 & 5 \\ \cancel{0} & \cancel{0} & \cancel{0} & \cancel{6} & \cancel{0} \\ 4 & 0 & 5 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \\ 1 & 2 & 4 & 5 \\ 1 & 1 & 2 & 5 \end{bmatrix}$$

$$6 \begin{bmatrix} 2 & 0 & 1 & 3 \\ 2 & 3 & 5 & 1 \\ \cancel{0} & \cancel{0} & \cancel{0} & \cancel{5} \\ 4 & 0 & 5 & 0 \end{bmatrix} = 6(-5) \begin{bmatrix} 2 & 0 & 1 \\ \cancel{2} & \cancel{3} & \cancel{5} \\ 4 & 0 & 5 \end{bmatrix} = 6 \cdot (-5) \cdot 3 \begin{bmatrix} 2 & 1 \\ 4 & 5 \end{bmatrix} = -90(10-4) = -540$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \\ 1 & 2 & 4 & 5 \\ 1 & 1 & 2 & 5 \end{bmatrix} \xrightarrow[-R_1]{-R_2} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 4 \end{bmatrix} \xrightarrow[-R_3]{-R_4} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 3 \end{bmatrix} \quad \det = 3$$

Note: You only need to do enough row reduction to get to triangular form. Any further row operations are wasted effort.

Example:

$$\begin{bmatrix} 2 & 3 & 7 & 11 \\ 2 & 9 & 12 & 35 \\ 4 & 18 & 10 & 0 \\ 6 & 21 & 17 & 15 \end{bmatrix} \xrightarrow[-R_2]{-R_1} \begin{bmatrix} 2 & 3 & 7 & 11 \\ 0 & 6 & 5 & 24 \\ 0 & 0 & -14 & -70 \\ 0 & 0 & 0 & 4 \end{bmatrix} \xrightarrow[-R_3-R_1]{-R_3-R_1}$$

$$\det = 2 * 6 * (-14) * 4$$