Name: ______________________________

Math 2250, Fall 2011, Quiz 7 **October 21, 2011** **R. Bruner**

Here is a matrix A and its reduced row echelon form:

$$\begin{bmatrix} 1 & 2 & 5 & 1 & 3 \\ -2 & 2 & 4 & 4 & 3 \\ -4 & 1 & 1 & 5 & 1 \\ 2 & 1 & 3 & -1 & 3 \end{bmatrix} \longrightarrow \begin{array}{c} \begin{array}{ccccc} * & * & & & * \end{array} \\ \begin{bmatrix} 1 & 0 & 1/3 & -1 & 0 \\ 0 & 1 & 7/3 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Give bases for Null(A) and Col(A).
Do the dimensions of these subspaces add up to the correct total?

$$\underline{\text{Basis for } Col(A)} = \left\{ \begin{bmatrix} 1 \\ -2 \\ -4 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 3 \\ 1 \\ 3 \end{bmatrix} \right\}$$

corresponding to pivot columns 1, 2 and 5.

$$\underline{\text{Basis for } Null(A)} = \left\{ \begin{bmatrix} -1/3 \\ -7/3 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 0 \\ 1 \\ 0 \end{bmatrix} \right\} \quad \text{since}$$

$$\left\{ \begin{aligned} x_1 &= \tfrac{-1}{3} x_3 + x_4 \\ x_2 &= \tfrac{-7}{3} x_3 - x_4 \\ x_3 &= x_3 \\ x_4 &= \quad\quad\; x_4 \\ x_5 &= 0 \end{aligned} \right\} \quad \text{if } \begin{bmatrix} x_1 \\ \vdots \\ x_5 \end{bmatrix} \text{ is in the Null space.}$$

$$\dim(Col(A)) + \dim(Null(A))$$

$$= \quad 3 \quad + \quad 2$$

$$= 5$$

so Yes, the dimensions add up to the correct total, the dimension of the domain.