

Compute the inverse of

$$\begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix}$$

if it is invertible. If not, find an element in its kernel (null space).

$$\left[\begin{array}{ccc|ccc} -1 & 1 & 1 & 1 & 0 & 0 \\ 1 & -1 & 1 & 0 & 1 & 0 \\ 1 & 1 & -1 & 0 & 0 & 1 \end{array} \right] \xrightarrow{\begin{array}{l} * -1 \\ +R_1 \\ +R_1 \end{array}} \left[\begin{array}{ccc|ccc} 1 & -1 & -1 & -1 & 0 & 0 \\ 0 & 0 & 2 & 1 & 1 & 0 \\ 0 & 2 & 0 & 0 & 0 & 1 \end{array} \right] \xrightarrow{\begin{array}{l} * 1/2 \\ * 1/2 \end{array}}$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & -1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1/2 & 1/2 & 0 \\ 0 & 1 & 0 & 1/2 & 0 & 1/2 \end{array} \right] \xrightarrow{+R_2+R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 1/2 & 1/2 \\ 0 & 1 & 0 & 1/2 & 0 & 1/2 \\ 0 & 0 & 1 & 1/2 & 1/2 & 0 \end{array} \right]$$

$$\begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix}^{-1} = \begin{bmatrix} 0 & 1/2 & 1/2 \\ 1/2 & 0 & 1/2 \\ 1/2 & 1/2 & 0 \end{bmatrix}$$