Name: KEY to Solutions

Math 2250, Fall 2011, Quiz 5 October 7, 2011 R. Bruner

Suppose that $T\begin{bmatrix}3\\2\end{bmatrix} = \begin{bmatrix}5\\2\end{bmatrix}$ and $T\begin{bmatrix}4\\3\end{bmatrix} = \begin{bmatrix}2\\1\end{bmatrix}$.

1. Express $\begin{bmatrix}1\\0\end{bmatrix}$ as a linear combination of $\begin{bmatrix}3\\2\end{bmatrix}$ and $\begin{bmatrix}4\\3\end{bmatrix}$.

2. Use that to compute $T\begin{bmatrix}1\\0\end{bmatrix}$.

3. Write the matrix form of T.

---

1. $$\left[\begin{array}{cc|cc}3 & 4 & 1 & 0\\ 2 & 3 & 0 & 1\end{array}\right] \xrightarrow{-R_2} \left[\begin{array}{cc|cc}1 & 1 & 1 & -1\\ 2 & 3 & 0 & 1\end{array}\right] \xrightarrow{-2R_1} \left[\begin{array}{cc|cc}1 & 1 & 1 & -1\\ 0 & 1 & -2 & 3\end{array}\right] \xrightarrow{-R_2}$$

$$\left[\begin{array}{cc|cc}1 & 0 & 3 & -4\\ 0 & 1 & -2 & 3\end{array}\right] \quad \text{so} \quad \boxed{\begin{bmatrix}1\\0\end{bmatrix} = 3\begin{bmatrix}3\\2\end{bmatrix} - 2\begin{bmatrix}4\\3\end{bmatrix}} \quad 3$$

$$\text{and} \quad \begin{bmatrix}0\\1\end{bmatrix} = -4\begin{bmatrix}3\\2\end{bmatrix} + 3\begin{bmatrix}4\\3\end{bmatrix}$$

2. $$T\begin{bmatrix}1\\0\end{bmatrix} = 3\,T\begin{bmatrix}3\\2\end{bmatrix} - 2\,T\begin{bmatrix}4\\3\end{bmatrix} = 3\begin{bmatrix}5\\2\end{bmatrix} - 2\begin{bmatrix}2\\1\end{bmatrix} = \boxed{\begin{bmatrix}11\\4\end{bmatrix}} \quad 3$$

3. $$T\begin{bmatrix}0\\1\end{bmatrix} = -4\begin{bmatrix}5\\2\end{bmatrix} + 3\begin{bmatrix}2\\1\end{bmatrix} = \begin{bmatrix}-14\\-5\end{bmatrix}$$

$$\boxed{T = \begin{bmatrix}11 & -14\\ 4 & -5\end{bmatrix}} \qquad 4 = 2 + 2$$

Check: $$\begin{bmatrix}11 & -14\\ 4 & -5\end{bmatrix}\begin{bmatrix}3\\2\end{bmatrix} = \begin{bmatrix}33-28\\ 12-10\end{bmatrix} = \begin{bmatrix}5\\2\end{bmatrix}$$

and $$\begin{bmatrix}11 & -14\\ 4 & -5\end{bmatrix}\begin{bmatrix}4\\3\end{bmatrix} = \begin{bmatrix}44-42\\ 16-15\end{bmatrix} = \begin{bmatrix}2\\1\end{bmatrix}$$