Name: KEY

Math 2250, Fall 2011, Quiz 3 **September 23, 2011** **R. Bruner**

Let $\mathbf{a}_1 = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$, $\mathbf{a}_2 = \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}$ and $\mathbf{a}_3 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ and $\mathbf{a}_4 = \begin{bmatrix} 4 \\ 0 \\ 4 \end{bmatrix}$.

1. Show that $\{\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3, \mathbf{a}_4\}$ is linearly dependent by finding a nontrivial linear combination of the vectors which is 0.
2. Find a linearly independent subset of them which contains as many vectors as possible.
3. Express the remaining vector (or vectors) as a linear combination of your linearly independent subset.

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1. $c_1 a_1 + c_2 a_2 + c_3 a_3 + c_4 a_4 = 0 \iff \begin{bmatrix} 1 & 2 & 1 & 4 \\ 1 & 2 & 1 & 0 \\ -1 & 2 & 0 & 4 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} = 0$ so we now reduce to find a solution.

$$\begin{bmatrix} 1 & 2 & 1 & 4 \\ 1 & 2 & 1 & 0 \\ -1 & 2 & 0 & 4 \end{bmatrix} \xrightarrow[+R_1]{-R_1} \begin{bmatrix} 1 & 2 & 1 & 4 \\ 0 & 0 & 0 & -4 \\ 0 & 4 & 1 & 8 \end{bmatrix} \xrightarrow[+2R_2]{+R_2,\ *-1/4} \begin{bmatrix} 1 & 2 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 4 & 1 & 0 \end{bmatrix} \xrightarrow[*\frac{1}{4}]{-\frac{1}{2}R_3} \begin{array}{c} \begin{array}{cccc} * & * & & * \end{array} \\ \begin{bmatrix} 1 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 1/4 & 0 \end{bmatrix} \end{array}$$

$\leftarrow$ pivot columns

$$x_1 + \tfrac{1}{2}x_3 = 0 \qquad x_2 + \tfrac{1}{4}x_3 = 0 \qquad x_4 = 0$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = x_3 \begin{bmatrix} -1/2 \\ -1/4 \\ 1 \\ 0 \end{bmatrix}$$

$$-\frac{1}{2}\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} - \frac{1}{4}\begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -\frac{1}{2} - \frac{1}{2} + 1 \\ -\frac{1}{2} - \frac{1}{2} + 1 \\ \frac{1}{2} - \frac{1}{2} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

2. Pivots in columns 1,2,4 $\Rightarrow \{a_1, a_2, a_4\}$ is a maximal linearly independent subset.

3. $\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} + \frac{1}{4}\begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}$ from the linear combinatin in problem #1.