Name: ______________________________

**Math 2250, Fall 2011, Quiz 14** **December 12, 2011** **R. Bruner**

Typo: This should be $x^2+2x+1$

Both

$$\mathcal{B} = \{x^2 + x, x^2 + 2x + 2, x^3 - x^2 - 2x\}$$

and

$$\mathcal{C} = \{x + 1, x^2 - 1, x^3 + 1\}$$

are bases for the space $N = \{p \in P_3 \mid p(-1) = 0\}$ of cubic polynomials whose value at $-1$ is 0. The coordinate change matrix $P_{\mathcal{C}\leftarrow\mathcal{B}}$ which converts from $\mathcal{B}$ coordinates to $\mathcal{C}$ coordinates is

$$P_{\mathcal{C}\leftarrow\mathcal{B}} = \begin{bmatrix} 1 & 2 & -2 \\ 1 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}.$$

1. Find the inverse matrix $P_{\mathcal{B}\leftarrow\mathcal{C}}$.
2. Use it to write $x + 1$, $x^2 - 1$ and $x^3 + 1$ as linear combinations of elements of $\mathcal{B}$.

(1) Proceeding as if there were no typo, to invert the given matrix we would get

$$P_{\mathcal{B}\leftarrow\mathcal{C}} = P_{\mathcal{C}\leftarrow\mathcal{B}}^{-1} = \begin{bmatrix} 1 & 2 & -2 \\ 1 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} -1 & 2 & 0 \\ 1 & -1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

(2) Interpreting the columns of this inverse we would get

$$x+1 = -(x^2+x) + (x^2+2x+2)$$

$$x^2-1 = 2(x^2+x) - (x^2+2x+2)$$

$$x^3+1 = (x^2+2x+2) + (x^3-x^2-2x)$$

but these lin. comb.'s are actually

$x+2$

$x^2-2$

$x^3+2$

since $\mathcal{B}$ is a basis of a <u>different</u> subspace, The subspace of cubics $p(x)$ satisfying

$p(0) = 2p(-1)$ !